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Geophysical Wavelet Library: Applications of the Continuous Wavelet Transform to the Polarization and Dispersion Analysis of Signals

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Abstract *In the present paper, we propose a software package developed by the authors and based on the continuous wavelet transform. This package allows to perform the direct and inverse continuous wavelet transform, 2C and 3C polarization analysis and filtering, modeling the dispersed and attenuated wave propagation in the time-frequency domain and optimization in signal and wavelet domains. The aim of these operations is to extract velocities and attenuation parameters from a seismogram. The novelty of this package is that we incorporate the continuous wavelet transform into the library where the kernel is the time-frequency polarization and dispersion analysis. This library has a wide range of potential applications and can be particularly suitable in geophysical problems that we illustrate with the analysis of synthetic, geomagnetic or real seismic data.*

Keywords: Wavelet transform, signal processing, dispersion, polarization, MATLAB

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1 Introduction

Frequency-dependent measurements or time-frequency analysis (TFR) offers additional insight and performance for any applications where Fourier techniques have been used. This analysis consists of examining the variation of the frequency content of a signal with time and is particularly suitable in geophysical applications. We can emphasize three directions to use TFR for the analysis of geophysical data. As the first very helpful tool, time-frequency representations can be incorporated in the polarization analysis [1]–[5]. It is also possible to model dispersive and dissipative wave propagation in the time-frequency domain [6]. Finally, the time-frequency analysis is suitable for an estimate of the phase velocity (the group velocity) and the attenuation coefficient [7]–[9].

The continuous wavelet transform (CWT) gives a suitable general framework for solving these types of problems; this approach is powerful and elegant, but is not the only available for the practical applications. Other TFR methods such as the Gabor transform, the S-transform [2] or bilinear transforms like the Wigner-Ville [8] or smoothed Wigner-Ville transform can be used as well. The relative performance of time-frequency analysis from different TFR approaches is primarily controlled by the frequency resolution capability.

This article summarizes our previous works aimed to the polarization and dispersion analysis of signals in the wavelet domain and offers the Geophysical Wavelet Library (GWL) — a new free software package based on CWT and having the following key features:

1. object-based implementation of main data types like a vector, an axis, a matrix, a multi-channel signal, a multi-channel wavelet spectrum;

2. object-based implementation of main mathematical objects like Morlet and Cauchy wavelets, some function approximations, 2C and 3C polarization parameters and dispersion parameters;

3. command line and MATLAB interface for transformations like the Fourier transform, the direct and inverse CWT, 2C and 3C polarization transforms, linear and nonlinear deformations of a wavelet spectrum;

4. command line interface for the optimization in signal and wavelet domains using the algorithm of Levenberg-Marquardt optimization with the aim to extract velocities and attenuation parameters from a seismogram;

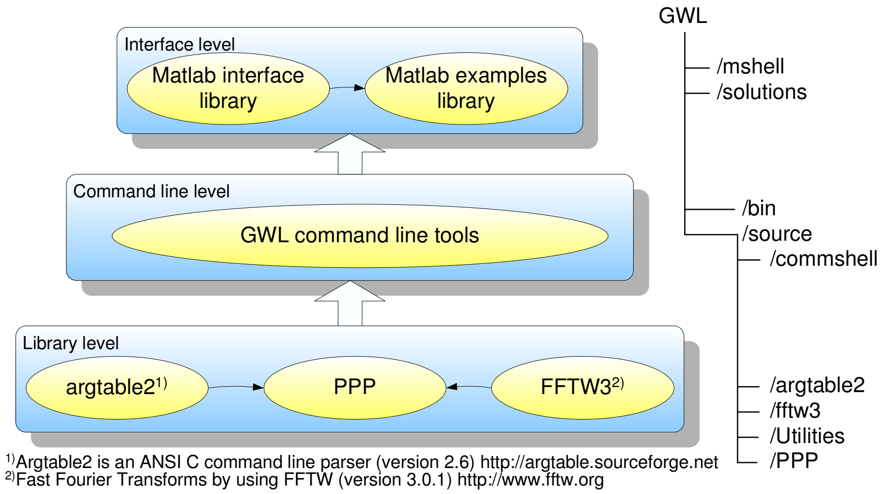

Figure 1: Structure of GWL

5. data import from tabulated and plain ASCII files as well as data export into ASCII files.

One can find and download in internet a great number of other free or commercial wavelet-based software. Unfortunately, all of wavelet based software packages presented in internet do not contain feature tailored for geophysical problems. With GWL, we try to address these limitations and incorporate CWT into the library where the kernel is the time-frequency polarization and dispersion analysis. The main purpose of this article is to show not only mathematical aspects of this problem, but also some peculiarities of implementation.

2 GWL structure and implementation technology

GWL includes three logical levels: the library level, the level of command line tools and the interface level as shown in Fig. 1.

The main part of the library level is a C++ hierarchical object library called PPP. Command line level is a set of independent C++ modules, which are based on PPP library and provide a command line interface for all methods implemented in GWL. After the compilation, we obtain a set of executable modules placed in GWL/bin directory. To perform a calculation, we run certain modules from GWL/bin directory in the appointed order. The calculation parameters have to be given by command line. The data exchange between different modules is implemented by the data files with a binary stream format. After the calculation process is finished, we obtain the collection of ASCII or binary files with calculation results.

We also developed an especial MATLAB package placed on interface level of GWL in the directory GWL/mshell. This package allows us to read all binary formats supported in GWL and plot GWL objects using high-level subroutines based on the standard MATLAB plotting commands. To show the application possibilities of the CWT for dispersion and polarization analysis of synthetic, geomagnetic and real seismic data, we stored many examples into GWL/solutions directory.

3 Continuous wavelet transform

The wavelet transform of a real or complex signal $S(t) \in L^2(\mathbb{R})$ with respect to a real or complex mother wavelet $g(t)$ is the set of L^2–scalar products of all dilated and translated wavelets with an arbitrary signal to be analyzed [10]:

$$\mathcal{W}_g S(t,a) = \langle g_{t,a}, S\rangle,$$
$$\mathcal{W}_g S(t,a) = \int\limits_{-\infty}^{+\infty} \frac{1}{a} g^*\Big(\frac{\tau - t}{a}\Big) S(\tau)\, d\tau, \qquad (1)$$

where $g_{t,a} = \frac{1}{a} g((\tau - t)/a)$ is generated from $g(t)$ through dilation $a \in \mathbb{R}$ and translation $t \in \mathbb{R}$. If we select a wavelet with a unit central frequency, it is possible to obtain the physical frequency directly by taking the inverse of the scale: $f = 1/a$. This approach is implemented in the module `gwlCwt` with the parameter `--wttype=0`.

The procedure (1) is very slow and therefore cannot be used for long signals. We can construct a more effective algorithm if we note the fact that the wavelet transform can be expressed in terms of the Fourier transform $\hat{S}(\zeta)$ of a signal $S(t)$ as

$$\mathcal{W}_g S(t,f) = \int_{-\infty}^{+\infty} \hat{g}^*(\zeta/f) e^{2\pi i t\zeta} \hat{S}(\zeta)\, d\zeta. \quad (2)$$

This fast approach is implemented in the module `gwlCwt` as well, but with the parameter `--wttype=1`.

The choice of the wavelet and the initialization of the wavelet parameter are represented in the module `gwlCwt` using the parameters `--wavelet` and `--wavepar`. In this work, we implemented in GWL only two progressive wavelets: the complex Morlet wavelet and the complex Cauchy wavelet [10].

We implemented the inverse CWT in the module `gwlIwt`:

$$S(t) = \mathcal{M}_h \mathcal{W}_g S(t,f), \quad (3)$$

$$S(t) = \frac{1}{C_{g,h}} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h\left(f(t-\tau)\right) \mathcal{W}_g S(\tau,f)\, d\tau\, df,$$

where $h(t)$ is the wavelet used for the inverse wavelet transform $\mathcal{M}_h$. In the general case, this wavelet can be different from the analyzing wavelet $g(t)$. One can choose for the inverse CWT the δ-function as the wavelet $h(t)$ (`gwlIwt --wavelet=delta`), which gives us a rather simple and fast reconstruction formula

$$S(t) = \mathcal{M}_h \mathcal{W}_g S(t,f) = \frac{1}{C_{g,\delta}} \int_{-\infty}^{+\infty} \mathcal{W}_g S(t,f)\, \frac{df}{f}. \quad (4)$$

$C_{g,h}$ in eq. (3) and $C_{g,\delta}$ in eq. (4) are the normalization coefficients related to the direct and inverse mother wavelets. We need to calculate this coefficient not for all applications; therefore, we implemented the parameter `--ampl` in the module `gwlIwt`, which defines the normalization mode of the inverse signal.

4 Polarization properties and polarization filtering

GWL contains several modules for the polarization analysis and polarization filtering of two-component and three-component signals:

1. The module `gwlET2D` with the input parameter `--type=complex` performs the calculation of polarization attributes of a two-component signal in the wavelet domain [4]:

$$\begin{aligned} R(t,f) &= \tfrac{1}{2}|\mathcal{W}_g^+ Z(t,f)| + |\mathcal{W}_g^- Z(t,f)|, \\ r(t,f) &= \tfrac{1}{2}||\mathcal{W}_g^+ Z(t,f)| - |\mathcal{W}_g^- Z(t,f)||, \\ \theta(t,f) &= \tfrac{1}{2}\arg[\mathcal{W}_g^+ Z(t,f) \mathcal{W}_g^- Z(t,f)], \\ \Delta\phi(t,f) &= \arg\left(\frac{\mathcal{W}_g^+ Z(t,f) + \mathcal{W}_g^- Z(t,f)^*}{\mathcal{W}_g^+ Z(t,f) - \mathcal{W}_g^- Z(t,f)^*}\right). \end{aligned} \quad (5)$$

where R is the semi-major axis, r is the semi-minor axis, θ is the tilt angle, which is the angle of the semi-major axis with the horizontal axis and $\Delta\phi$ is the phase difference between two orthogonal signal components $S_x(t)$ and $S_z(t)$. The complex signal $Z(t) = S_x(t) + iS_z(t)$.

2. The module `gwlET2DFilter` implements a filtering algorithm based on the instantaneous attributes (5) by a combination of constraints posed on the range of the reciprocal ellipticity $\rho(t,f) = r(t,f)/R(t,f)$ and the tilt angle $\theta(t,f)$:

$$Z^f(t) = \mathcal{M}_h \mathcal{E}_{\rho\theta} \mathcal{W}_g Z(t,f), \quad (6)$$

$$\mathcal{E}_{\rho\theta}(t,f) = \begin{cases} 0, & \rho(t,f) \notin P_\rho \text{ or } \theta(t,f) \notin P_\theta, \\ \mathcal{W}_g Z(t,f), & \text{otherwise,} \end{cases}$$

where $\mathcal{E}_{\rho\theta}$ is the filter operator of the wavelet-spectrum. The sets P_ρ and P_θ define the range of ρ and θ, which are kept in the filtered signal.

3. The modules `gwlET3D --type=morozov` and `gwlET3DFilter` extended the method of [11] to the wavelet domain in order to use the instantaneous attributes for filtering and wavefield separation for any number of components [3]:

$$\begin{aligned} \psi_0(t,f) &= \tfrac{1}{2}\arg\left[A(t,f) + \varepsilon B(t,f)\right] + \pi n, \\ \mathbf{R}(t,f) &= \Re[e^{-i\psi_0(t,f)} \mathcal{W}_g \mathbf{S}(t,f)], \\ \mathbf{r}(t,f) &= \Re[e^{-i(\psi_0(t,f)+\pi/2)} \mathcal{W}_g \mathbf{S}(t,f)], \\ A(t,f) &= \tfrac{1}{2}\textstyle\sum_k \mathcal{W}_g S_k(t,f)^2, \\ B(t,f) &= \tfrac{1}{2}(\textstyle\sum_k \mathcal{W}_g S_k(t,f))^2, \end{aligned} \quad (7)$$

where $\mathcal{W}_g \mathbf{S}(t,f)$ is the component-wise calculated progressive wavelet spectra ($f > 0$, wavelet is progressive) of the multicomponent signal $\mathbf{S}(t) = [S_x(t), S_y(t), S_z(t)]$ and $n \in \mathbb{N}$ is any integer number.

4. Unfortunately, there is no mathematically exact a priori definition for the instantaneous polarization attributes of a multicomponent signal. The above described method is restricted by design to the characterization of an ellipse. In more general terms, particle motions captured with three-component recordings can be characterized by a polarization ellipsoid which can be approximated using the covariance method [12]. The module `gwlET3DFilter --type=acovar` extends the covariance method to the time-frequency domain using an adaptive approach to select an analyzed time window, which is derived from an averaged instantaneous frequency $\Omega(t,f)$ of the multicomponent record [13]. In this approach, the entries of the cross-energy matrix $\mathbf{M}(t,f) = [M_{km}(t,f)]$ can be calculated as

$$\begin{gathered} M_{km}(t,f) = |\mathcal{W}_g S_k(t,f)||\mathcal{W}_g S_m(t,f)| \cdot \\ \{\mathrm{sinc}(\Gamma^-_{km}(t,f)) \cos(A^-_{km}(t,f)) + \\ \mathrm{sinc}(\Gamma^+_{km}(t,f)) \cos(A^+_{km}(t,f))\} - \mu_{km}\mu_{mk}, \\ \Gamma^{\pm}_{km}(t,f) = \tfrac{\Delta t_{km}(t,f)}{2}(\Omega_k(t,f) \pm \Omega_m(t,f)), \\ A^{\pm}_{km}(t,f) = \arg \mathcal{W}_g S_k(t,f) \pm \\ \arg \mathcal{W}_g S_m(t,f), \\ \Delta t_{km}(t,f) = \tfrac{4\pi n}{\Omega_k(t,f)+\Omega_m(t,f)}, \\ \mu_{km} = \Re[\mathcal{W}_g S_k(t,f)]\mathrm{sinc}(\tfrac{\Delta t_{km}(t,f)\Omega_k(t,f)}{2}), \\ n \in \mathbb{N}, \quad k,m = x,y,z, \end{gathered} \tag{8}$$

where $\mathrm{sinc}(x)$ indicates the sine cardinal function. When the instantaneous frequencies are the same for all components, this method produces the same results as those by [11] in terms of polarization parameters.

5 Modeling of wave propagation using a diffeomorphism in wavelet space

In order to analyze the dynamical behavior of multivariate signals using the continuous wavelet transforms, it can be interesting to investigate a diffeomorphic deformation of the wavelet space. These deformations establish algebra of wavelet pseudodifferential operators acting on signals:

1. Let us assume that frequency-dependent wavenumber $k(t)$ and attenuation $\alpha(f)$ are slowly varying with regard to the frequency range of the mother wavelet. We also assume that the attenuation shows nearly linear frequency dependence. In such a case, $\alpha'(f) \sim 0$ and the asymptotic propagator in the wavelet space has the form [6]:

$$\begin{gathered} \mathcal{W}_g S_m(t,f) = \mathcal{D}_{\mathcal{W}} \mathcal{W}_g S_k(t,f) = \\ e^{-\alpha(f)D} e^{-i\psi_1(f)} \mathcal{W}_g S_k\left(t - k'(f)D, f\right), \\ \mathcal{W}_g T_{mk}(t,f) = \mathcal{D}_{\mathcal{CW}} \mathcal{W}_g T_{rr}(t,f) = \\ e^{-\alpha(f)D_{mk}} e^{-i\psi_1(f)} \mathcal{W}_g T_{rr}\left(t - k'(f)D, f\right), \\ \psi_1(f) = 2\pi(k(f) - fk'(f))D + 2\pi n, \end{gathered} \tag{9}$$

where $\hat{T}_{mk}(f)$ is the cross-correlation in the Fourier domain, when we take a trace $S_r(t)$ as reference and two other traces $S_m(t)$ at a distance D_m and $S_k(t)$ at a distance D_k (distance with respect to the position of the reference). $\hat{T}_{rr}(f) = |\hat{S}_r(f)|^2$, $D_{mk} = D_m + D_k$, $D = D_m - D_k$. $n \in \mathbb{N}$ is any integer number.

2. In the special case, with the assumption that the analyzing wavelet has a linear phase (for example, Morlet wavelet), the approximation (9.1) can be written in terms of the phase $C_p(f) = f/k(f)$ and group $C_g(f) = 1/k'(f)$ velocities as [16]:

$$\begin{gathered} \mathcal{W}_g S_m(t,f) = \exp\left(-\alpha(f)D\right) \cdot \\ \left|\mathcal{W}_g S_k\left(t - \tfrac{D}{C_g(f)}, f\right)\right| \cdot \\ \exp\left[i \arg \mathcal{W}_g S_k\left(t - \tfrac{D}{C_p(f)} - \tfrac{n}{f}, f\right)\right]. \end{gathered} \tag{10}$$

This behavior is demonstrated in Fig. 2, where we consider a synthetic signal $S_1(t)$. In this example, we use phase and group velocities, which are not based on a physical model. These frequency-dependent velocities are shown in Figs. 2c,d. We perform the propagation of signal $S_1(t)$ using the equation (9.1) and obtain a propagated counterpart $S_2(t)$. The gray-scaled images in Fig. 2 show the absolute values and phases of wavelet spectra $\mathcal{W}_g S_1(t,f)$ and $\mathcal{W}_g S_2(t,f)$. We see that the deformations of images labeled as $|\mathcal{W}_g S_2(t,f)|$ and $\arg \mathcal{W}_g S_2(t,f)$ agree in general with velocities curves $C_g(f)$ and $C_p(f)$ accordingly, but have small distinctions that demonstrate the asymptotic properties of the equation (10).

This procedure (9) is implemented in the module `gwlDiffeoDisp` with the parameter `--prop=1`, while the procedure (10) can be executed with the parameter `--prop=2`. Note that the frequency-dependent wavenumber and attenuation in the propagation models (9)-(10)

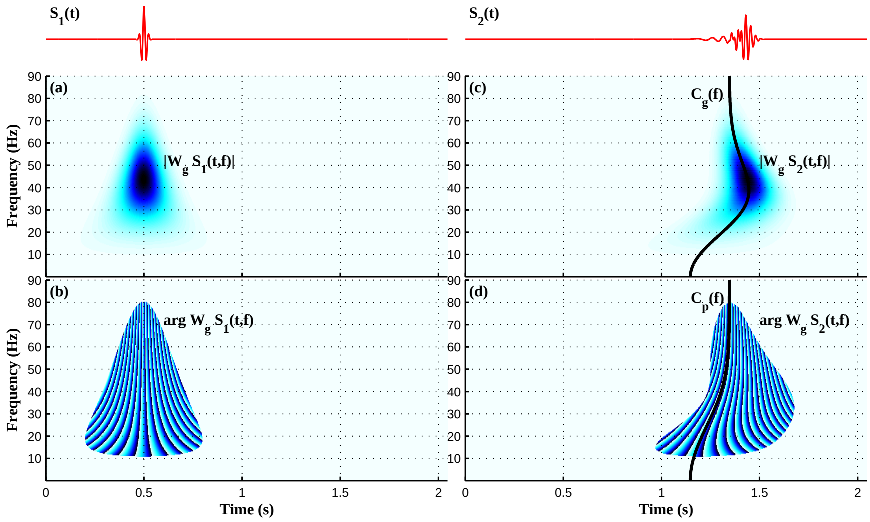

Figure 2: Propagated synthetic signal and its wavelet transform: (a),(c) are the power (absolute value squared) of the wavelet coefficients and (b),(d) are the corresponding phase images. The lines in (c) and (d) show frequency-dependent group and phase velocities used in propagation model

are independent and therefore do not satisfy the causality constraint.

3. In order to satisfy the causality constraint, instead of the assumption $\alpha'(f) \sim 0$, a special wavelet like Cauchy wavelet can be used, which allows to derive a relationship between the wavelet transforms of signals observed at two different stations in terms of complex wavenumber $\mathbb{K}(f) = 2\pi k(f) - i\alpha(f)$:

$$\begin{gathered} \mathcal{W}_g S_m(t,f) = \mathcal{D}_{\mathcal{CC}} \mathcal{W}_g S_k(t,f) = \\ \frac{1}{f_\alpha^{p-1}(f)} \exp\left(-i[\mathbb{K}(f) - f\mathbb{K}'(f)]D\right) \cdot \\ \mathcal{W}_g S_k\left(t - \frac{D}{2\pi}\Re\mathbb{K}'(f), \frac{f}{f_\alpha(f)}\right), \\ f_\alpha(f) = 1 - \frac{fD}{p-1}\Im\mathbb{K}'(f). \end{gathered} \quad (11)$$

This propagator is implemented in the module `gwlDiffeoDisp --prop=3`.

6 Estimate of the phase and group velocity and the attenuation

Equation (10) allows us to formulate the idea how the frequency-dependent phase velocity can be obtained using the wavelet spectra' phases of source and propagated signals:

1. Using the correlations between two spectra, we can perform the "frequency-velocity" analysis in the module `gwlTransFK` on the analogy of frequency-wavenumber method for a seismogram $S_k(t), k = 1, N$. The main part of this analysis consists of the calculation of correlation spectrum $\mathbf{M}(f, c)$ as follows:

$$\mathbf{M} = \int_{t_{min}}^{t_{max}} \left| \sum_{k,m} A_k(\tau, f)\, A_m^*\left(\tau - \frac{D_{km}}{c}, f\right) \right| d\tau,$$

or

$$\mathbf{M} = \int_{t_{min}}^{t_{max}} \left| \sum_{k,m} e^{iB_k(\tau,f)} e^{-iB_m\left(\tau - \frac{D_{km}}{c}, f\right)} \right| d\tau,$$

$$A_k(\tau, f) = \frac{\mathcal{W}_g S_k(\tau, f)}{|\mathcal{W}_g S_k(\tau, f)|},$$

$$B_k(\tau, f) = \arg \mathcal{W}_g S_k(\tau, f).$$

where $[t_{min}, t_{max}]$ indicates the total time range for which the wavelet spectrum was calculated, $c \in [C_p^{min}, C_p^{max}]$ is an unbound variable corresponding to the phase velocity, A_k is a complex-valued wavelet phase and B_k is a real-valued wavelet phase.

2. For given parametrization of dispersion and attenuation functions, finding an acceptable set of parameters can be thought as an optimization problem:

$$\chi^2(\alpha(f, \mathbf{p}), k(f, \mathbf{q})) \rightarrow min, \;\; \mathbf{p} \in \mathbb{R}^P, \; \mathbf{q} \in \mathbb{R}^Q,$$

where P is the number of parameters used to model the attenuation $\alpha(f)$ and Q is the number of parameters used to model the wavenumber $k(f)$. $\mathbf{p}$ and $\mathbf{q}$ represent the vectors of parameters describing the attenuation and the wavenumber respectively. This cost function involves a propagator described in the previous section depending on the nature of the signal to be analyzed. In the following, we intend to discuss the different steps involved in our inversion algorithm.

- The first step will consist of seeking a good initial condition for $k(f)$ and $\alpha(f)$ by performing an image matching using the modulus of the wavelet transforms of a pair of traces (the module `gwlOptiSP --cmpl=3`):

$$\chi^2(\mathbf{p},\mathbf{q}) = \sum_{m,k} \int\int \left| |\mathcal{W}_g S_k(t,f)| - |\mathcal{D}_{\mathcal{W}}(\mathbf{p},\mathbf{q})\mathcal{W}_g S_m(t,f)| \right|^2 dt\, df. \tag{12}$$

- In the case when the observed signals consist of a mixture of different wave types and modes, a cascade of optimizations in the wavelet domain will be necessary in order to fully determine the dispersion and attenuation characteristics specific to each coherent arrival. Firstly, we perform the optimization on the modulus of the transforms in which case the attenuation for the specified event is derived (`gwlOptiSP --cmpl=3`):

$$\chi^2(\mathbf{p},\mathbf{q}) = \sum_{m,k} \int\int \left| |\mathcal{W}_g T_{mk}(t,f)| - |\mathcal{D}_{\mathcal{CW}}(\mathbf{p},\mathbf{q})\mathcal{W}_g T_{rr}(t,f)| \right|^2 dt\, df. \tag{13}$$

  Next, we perform an optimization involving the argument of the wavelet transforms which will finally provide the phase and group velocity curves of the analyzed coherent arrival (`gwlOptiSP --cmpl=4`):

$$\chi^2(\mathbf{p},\mathbf{q}) = \sum_{m,k} \int\int \left| \arg \mathcal{W}_g T_{mk}(t,f) - \arg \mathcal{D}_{\mathcal{CW}}(\mathbf{p},\mathbf{q})\mathcal{W}_g T_{rr}(t,f) \right|^2 dt\, df. \tag{14}$$

  This optimization can be repeated to characterize each coherent arrival separately. Minimizing the cost function based on the cross-correlations is more advantageous for two reasons. On one hand, the effect of random noise is canceled, on the other, with geophones laid out symmetrically around the source in a seismic survey, cross-correlations of traces from seismic waves propagating in opposite directions can be combined in the optimization.

Since the dependence of the cost functions (12)-(14) on the parameters $\mathbf{p}$ and $\mathbf{q}$ is highly non-linear, each function may have several local minima. To obtain the global minimum that corresponds to the true parameters, a non-linear least-squares minimization method that proceeds iteratively from a reasonable set of initial parameters is required. In the present contribution, we use the Levenberg-Marquardt algorithm [14].

7 Conclusion

We propose a software package, which implements some methods of polarizations and dispersion analysis in wavelet-domain using the continuous wavelet transform:

1. Method for computing instantaneous attributes of 2-C signals in the time-frequency domain. The advantage of this method over previous techniques [15] is that both the time and frequency dependence of the attributes can be obtained and used for wave mode separation and filtering;

2. Extension of the polarization analysis technique for multicomponent data initially proposed in [11] into the time-frequency domain;

3. Method for the estimation of instantaneous polarization attributes based on an approximation to the covariance matrix and an extension of the adaptive covariance method to the time-frequency domain. The advantage of the proposed method over the standard method is that the length of the window size for the covariance computation is adaptively adjusted with the help of the instantaneous frequencies from the different components;

4. Some methods to establish a link between the continuous wavelet transform of a signal and its propagated counterpart in a dispersive and attenuating medium. The advantage of using the proposed propagator over traditional methods such as the Wigner-Ville or time frequency reassignment for dispersion curves estimates is that the full dispersion and dissipation characteristics are explicitly expressed and therefore can be easily extracted;

5. Approach to use this wavelet propagator in the method of "frequency-velocity" analysis in analogy to the classical frequency-wavenumber (f-k) analysis methods. Using this method, the determination of several mode branches is feasible;

6. Method of simultaneous computations of both phase and group velocity in the wavelet domain. The method owes its robustness to the fact that the minimization process involves not only the modulus but also the phase of the wavelet transform thus making it possible, in principle, to reconstruct the dispersed signal from the manipulated wavelet coefficients.

The mathematical aspects of all these methods have been separately published in [3, 4, 6, 9, 13, 16]. In this paper, we showed that all these methods could be logically combined into one library.

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