# Polarization Analysis of Pi2 Pulsations Using Continuous Wavelet Transform

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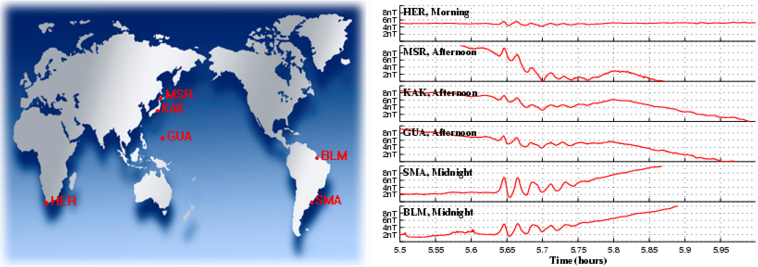
# WHAT IS PI2 PULSATION?

- Pi2 pulsation is defined as geomagnetic field variations with a period range of 40-150 sec. and an irregular waveform.
- Pi2 pulsations are observed more clearly on the nightside than on the dayside.

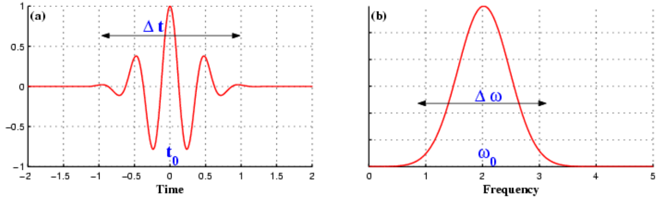


20 September 1995 event

(Nosé, M., *J. of Geoph. Res.*, **108**(A5), 1999)

# BUILDING BLOCKS: WAVELETS

Wavelet is a compromise between time and frequency localization



Two parameter family of wavelets

$$g_{t,a}(\tau) = T_t D_a g(\tau) = \frac{1}{a} g\left(\frac{\tau - t}{a}\right)$$

- scaling $D_a : g(\tau) \mapsto g(\tau/a)/a$
- translating $T_t : g(\tau) \mapsto g(\tau - t)$

# THE CONTINUOUS WAVELET TRANSFORM

$$\mathcal{W}_g S(t,a) = \tilde{g}_a * S(t) = \int\limits_{-\infty}^{+\infty} \frac{1}{a} g^* \left( \frac{\tau - t}{a} \right) S(\tau)\, d\tau$$

$$\mathcal{W}_g S(t,f) = \tilde{g}_f * S(t) = \int\limits_{-\infty}^{+\infty} f\, g^* \left( f(\tau - t) \right) S(\tau)\, d\tau$$

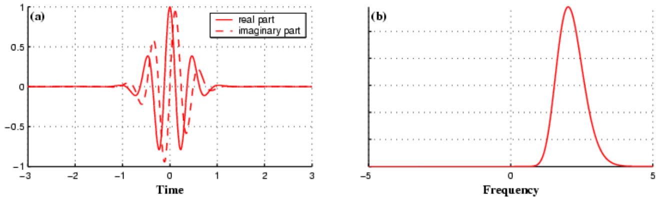
Interpretation as mathematical microscope or time-frequency analysis

- $a$: scale ↔ frequency $f = 1/a$
- $t$: position ↔ time
- $g$: optics ↔ filter
- $(\cdot)^*$ means the complex conjugate

# MOTHER WAVELETS

Complex Cauchy wavelet of power *p*

$$g(t) = \Gamma(p)\left(1 - \frac{2\pi i t}{p-1}\right)^{-p}, \quad \hat{g}(f) = f^{p-1} e^{-f(p-1)}$$



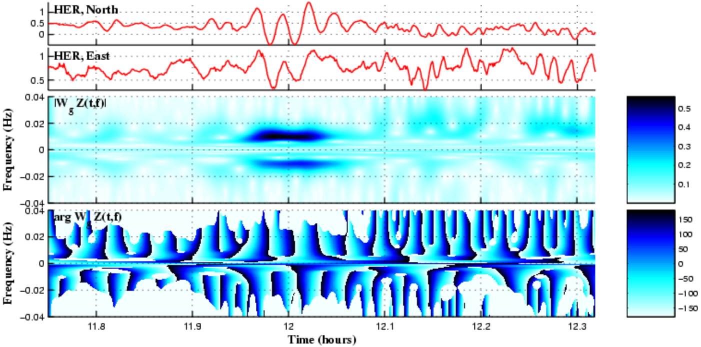
The Cauchy wavelet is progressive, i.e. its Fourier coefficients for negative frequencies are zero.

# COMPLEX SIGNAL

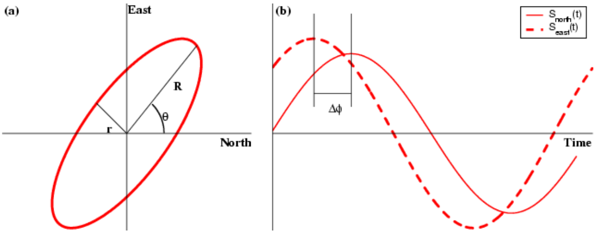
$S_{north}(t)$ and $S_{east}(t)$ are the north and east components of magnetic data. We construct a complex signal

$$Z(t) = S_{north}(t) + iS_{east}(t).$$

Progressive and regressive components of CWT



# ELLIPTICAL SIGNAL



An elliptically polarized rotating signal $Z(t)$ is described by:

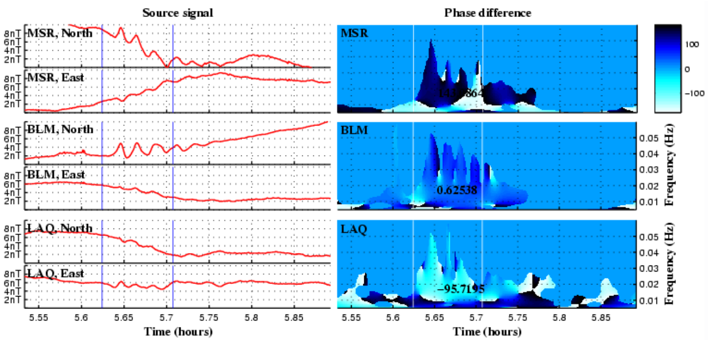
1. $R$: the semi-major axis $R \geq 0$
2. $r$: the semi-minor axis $r \geq 0$
3. $\rho = r/R$: the ellipticity ratio, $\rho \in [0, 1]$
4. $\Theta$: the tilt angle of the semi-major axis with the $X$ axis
5. $\Delta\phi$: the phase difference between components

# 2-C INSTANTANEOUS POLARIZATION PARAMETERS

Instantaneous polarization properties can be obtained for every time and frequency point in the time-frequency plane:

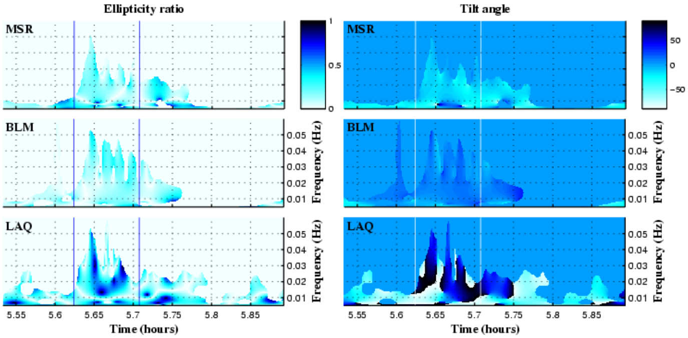
- Instantaneous frequencies: $\omega^{\pm}(t,f) = \pm\partial \arg \mathcal{W}_g^{\pm} Z(t,f)/\partial t$
- Local approximation around $t$: $\mathcal{W}_g Z(t+\tau, f) \simeq \mathcal{W}_g^{+} Z(t,f) e^{i\omega^{+}(t,f)\tau} + \mathcal{W}_g^{-} Z(t,f) e^{-i\omega^{-}(t,f)\tau}$
- Major half-axis $R(t,f) = |\mathcal{W}_g^{+} Z(t,f)| + |\mathcal{W}_g^{-} Z(t,f)|$
- Minor half-axis $r(t,f) = ||\mathcal{W}_g^{+} Z(t,f)| - |\mathcal{W}_g^{-} Z(t,f)||$
- Tilt angle $\Theta(t,f) = \arg[\mathcal{W}_g^{+} Z(t,f) \mathcal{W}_g^{-} Z(t,f)] \bmod \pi$
- Phase difference $\Delta\phi(t,f) = \arg\left(\frac{\mathcal{W}_g^{+} Z(t,f) + (\mathcal{W}_g^{-} Z(t,f))^*}{\mathcal{W}_g^{+} Z(t,f) - (\mathcal{W}_g^{-} Z(t,f))^*}\right) + \frac{\pi}{2}.$

# PHASE DIFFERENCE



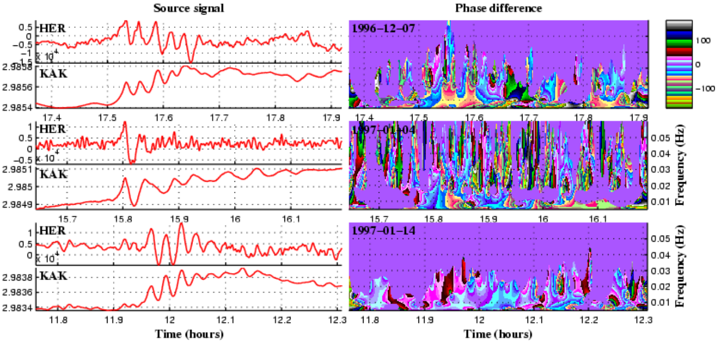
1. MSR has left-handed polarization (Lester et al., *J. Geophys. Res.*, **88**, 7958–7966, 1983)
2. BLM is characterized by linear polarization
3. LAQ has right-handed polarization (Nosé, M., *Earth Planets Space*, **51**, 23–32, 1999)

# ELLIPTICITY AND DIRECTION



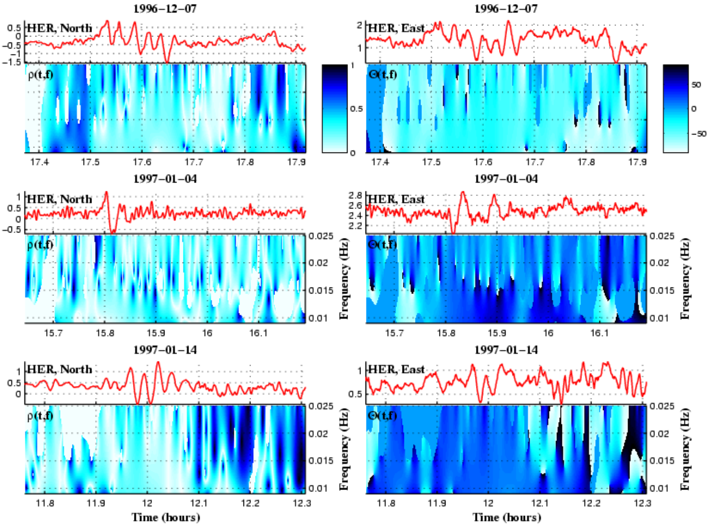
1. MSR: oscillations are dominant in north-east direction
2. BLM: oscillations are dominant in north-south direction
3. LAQ has elliptical polarization in east-west direction (Nosé, M., *Earth Planets Space*, **51**, 23–32, 1999)

# TWO STATIONS' ANALYSIS



1. The first two events: at the beginning the phase difference is small $\sim 0^\circ$, after 1-2 cycles the phase difference becomes negative value $\sim -45^\circ$
2. 14 January 1997: no phase differences between two stations during the event

# POLARIZATION AT HER STATION



# POLARIZATION FILTER

Filtration can be performed in the wavelet domain followed by an inverse wavelet transform to recover the filtered signal in the time domain.

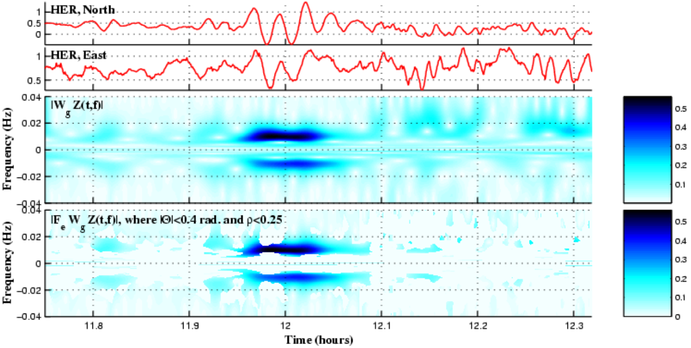
$$Z^f(t) = \mathcal{M}_g \mathcal{F}_e \mathcal{W}_g Z(t,f),$$

$$\mathcal{F}_e(t,f) = \begin{cases} \mathcal{W}_g Z(t,f) & \text{for } \rho(t,f) \in P_\rho \text{ and } \theta(t,f) \in P_\theta, \\ 0 & \text{otherwise,} \end{cases}$$

| Polarization | Filter | $P_\rho$ | $P_\theta$ |
|---|---|---|---|
| Linear and horizontally | $\mathcal{F}_{LH}$ | $\rho \notin [\rho_f, 1-\rho_f]$ | $\lvert\theta\rvert < \theta_f$ |
| Linear and vertically | $\mathcal{F}_{LV}$ | $\rho \notin [\rho_f, 1-\rho_f]$ | $\lvert\theta\rvert \geq \theta_f$ |
| Elliptic and horizontally | $\mathcal{F}_{EH}$ | $\rho \in [\rho_f, 1-\rho_f]$ | $\lvert\theta\rvert < \theta_f$ |
| Elliptic and vertically | $\mathcal{F}_{EV}$ | $\rho \in [\rho_f, 1-\rho_f]$ | $\lvert\theta\rvert \geq \theta_f$ |

$\theta_f$, $\rho_f$ are the parameters of polarizion filter.

# POLARIZATION AT HER STATION

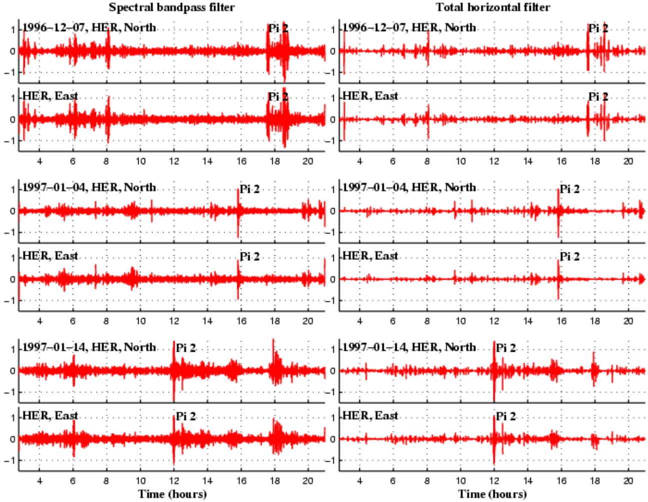


$Z(t)$ can be recovered from its wavelet transform:

$$Z(t) = \frac{1}{C_{g,m}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} m\left(f(t-\tau)\right) \mathcal{W}_g Z(\tau, f)\, d\tau\, df$$

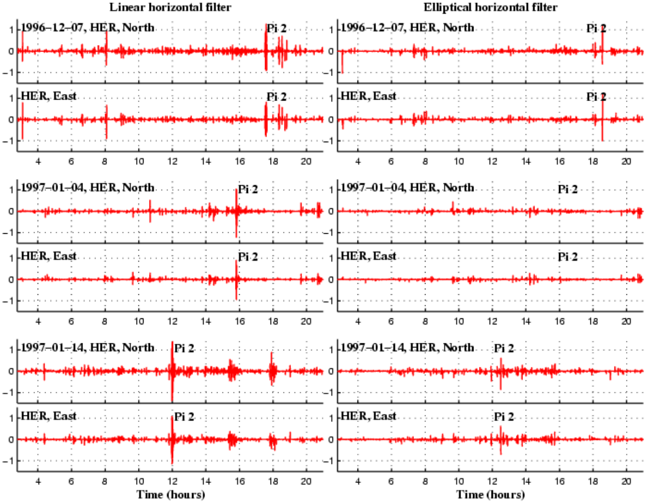
# FILTERING RESULT

Total horizontal filtering $\mathcal{F}_{LH} + \mathcal{F}_{EH}$, $\theta_f = 0.6$rad



# FILTERING RESULT

## Linear $\mathcal{F}_{LH}$ and elliptical $\mathcal{F}_{EH}$ filter, $\rho_f = 0.3$



# CONCLUSIONS

1. We propose a method for analysis of polarization properties of geomagnetic field data using the continuous wavelet transform.
2. Both time and frequency dependence of the polarization attributes can be obtained.
3. Visualization of these attributes in the time-frequency plane can be very helpful for the interpretation of geomagnetic pulsations.
4. Known polarization properties of Pi2 pulsation can be used for the construction of a polarization filter.
5. The polarization filter transmits the Pi2 pulsation clearer than Fourier band-pass filter.