Geophysical Wavelet Library

Applications of the Continuous Wavelet Transform to the Polarization and Dispersion Analysis of Signals

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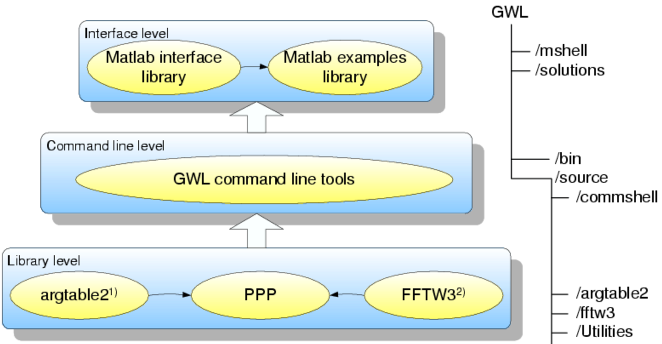
THE STRUCTURE OF GWL

This talk summarizes our previous works aimed to the polarization and dispersion analysis of signals in the wavelet domain and offers the Geophysical Wavelet Library (GWL) — a new free software package based on CWT.



[1)] Argtable2 is an ANSI C command line parser (version 2.6) http://argtable.sourceforge.net
[2)] Fast Fourier Transforms by using FFTW (version 3.0.1) http://www.fftw.org

THE CONTINUOUS WAVELET TRANSFORM

Time-frequency analysis / synthesis (CWT/ICWT):

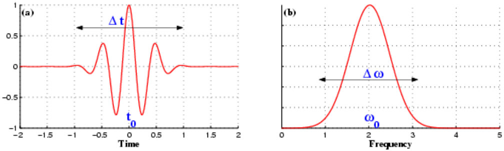
$$\mathcal{W}_g S(t,a) = \langle T_t D_a g, S \rangle = \int_{-\infty}^{+\infty} \frac{1}{a} g^* \left(\frac{\tau - t}{a} \right) S(\tau)\, d\tau$$

$$S(t) = \mathcal{M}_h \mathcal{W}_g S(t,a) = \frac{1}{C_{g,h}} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h \left(\frac{t-\tau}{a} \right) \mathcal{W}_g S(\tau,a) \frac{d\tau\, da}{a^2}$$

- g, h: optics $\leftrightarrow$ filter
- scaling $D_a : g(\tau) \mapsto g(\tau/a)/a$
- translating $T_t : g(\tau) \mapsto g(\tau - t)$
- a: scale $\leftrightarrow$ frequency $f = 1/a$
- t: position $\leftrightarrow$ time
- $C_{g,h}$ is the normalization coefficient

BUILDING BLOCKS: WAVELETS

Wavelet is a compromise between time and frequency localization



- Complex Morlet wavelet

$$g(t) = e^{2\pi i t} e^{-t^2/(2\sigma^2)}, \quad \hat{g}(\omega) = \sigma e^{-\sigma^2(\omega-2\pi)^2/2}$$

- Complex Cauchy wavelet of power p

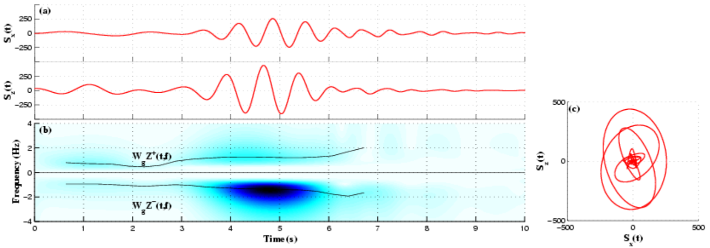
$$g(t) = \left(1 - \frac{2\pi i t}{p-1}\right)^{-p}, \quad \hat{g}(\omega) = \frac{(p-1)^p}{\sqrt{2\pi}(p-1)!} \left(\frac{\omega}{2\pi}\right)^{p-1} e^{-\omega(p-1)/(2\pi)}$$

TWO-COMPONENTS ELLIPTICAL SIGNAL

$S_z(t)$ and $S_x(t)$ are the north and east components of a 2-C signal. We construct a complex signal

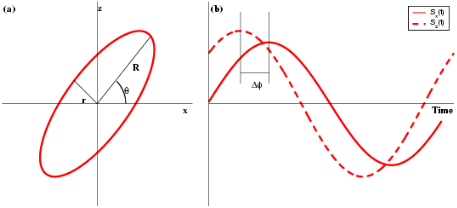
$$Z(t) = S_z(t) + iS_x(t).$$

Progressive and regressive components of CWT



2-C INSTANTANEOUS POLARIZATION PARAMETERS

An elliptically polarized rotating signal $Z(t)$ is described by (Diallo & Kulesh at al., *GEOPHYSICS*, **71**(3), 2006):



1. Major half-axis $R(t,f) = |\mathcal{W}_g^+ Z(t,f)| + |\mathcal{W}_g^- Z(t,f)|/2$
2. Minor half-axis $r(t,f) = ||\mathcal{W}_g^+ Z(t,f)| - |\mathcal{W}_g^- Z(t,f)||/2$
3. Tilt angle $\theta(t,f) = \arg[\mathcal{W}_g^+ Z(t,f) \mathcal{W}_g^- Z(t,f)]/2$
4. Phase difference
$$\Delta\phi(t,f) = \arg\left(\frac{\mathcal{W}_g^+ Z(t,f) + \mathcal{W}_g^- Z(t,f)^*}{\mathcal{W}_g^+ Z(t,f) - \mathcal{W}_g^- Z(t,f)^*}\right) \bmod \pi$$

2-C POLARIZATION FILTER

Filtration can be performed in the wavelet domain followed by an inverse wavelet transform to recover the filtered signal in the time domain.

$$Z^f(t) = \mathcal{M}_h \mathcal{E}_{\rho\theta} \mathcal{W}_g Z(t,f),$$

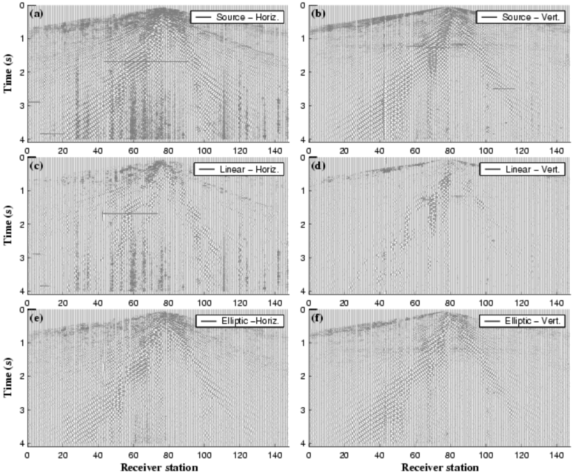
$$\mathcal{E}_{\rho\theta}(t,f) = \begin{cases} \mathcal{W}_g Z(t,f) & \text{for } \rho(t,f) \in P_\rho \text{ and } \theta(t,f) \in P_\theta, \\ 0 & \text{otherwise,} \end{cases}$$

Filter name	Filter	P_ρ	P_θ
Linear and horizontal	$\mathcal{E}_{LH}$	$\rho \notin [1-\rho_f,\ 1]$	$\lvert\theta\rvert < \theta_f$
Linear and vertical	$\mathcal{E}_{LV}$	$\rho \notin [1-\rho_f,\ 1]$	$\lvert\theta\rvert \geq \theta_f$
Elliptical and horizontal	$\mathcal{E}_{EH}$	$\rho \in [\rho_f,\ 1-\rho_f]$	$\lvert\theta\rvert < \theta_f$
Elliptical and vertical	$\mathcal{E}_{EV}$	$\rho \in [\rho_f,\ 1-\rho_f]$	$\lvert\theta\rvert < \theta_f$

θ_f, ρ_f are the parameters of polarizion filter.

REAL 2-C DATA FROM FORT WORTH BASIN U. S. A.

Separating the Rayleigh waves from the overall linearly polarized signals



3-C INSTANTANEOUS POLARIZATION PARAMETERS

We consider the cross-energy matrix $\mathbf{M}(t,f)$ with the eigenvalues and eigenvectors λ_k and $\mathbf{v}_k$

1. Major half-axis $\mathbf{R}(t,f) = \sqrt{\lambda_1(t,f)}\mathbf{v}_1(t,f)/\|\mathbf{v}_1(t,f)\|$
2. Minor half-axis $\mathbf{r}(t,f) = \sqrt{\lambda_3(t,f)}\mathbf{v}_3(t,f)/\|\mathbf{v}_3(t,f)\|$
3. 2^{nd} minor half-axis $\mathbf{r}_s(t,f) = \sqrt{\lambda_2(t,f)}\mathbf{v}_2(t,f)/\|\mathbf{v}_2(t,f)\|$

where (Kulesh & Diallo at al., *Geophys. J. Int*, **169**(4), 2007)

$$M_{km}(t,f) = |\mathcal{W}_g S_k(t,f)||\mathcal{W}_g S_m(t,f)|\{\mathrm{sinc}(\Gamma^-_{km}(t,f))\cos(A^-_{km}(t,f)) + \mathrm{sinc}(\Gamma^+_{km}(t,f))\cos(A^+_{km}(t,f))\} - \mu_{km}\mu_{mk},$$

$$\Gamma^\pm_{km}(t,f) = \frac{\Delta t_{km}(t,f)}{2}(\Omega_k(t,f) \pm \Omega_m(t,f)),$$

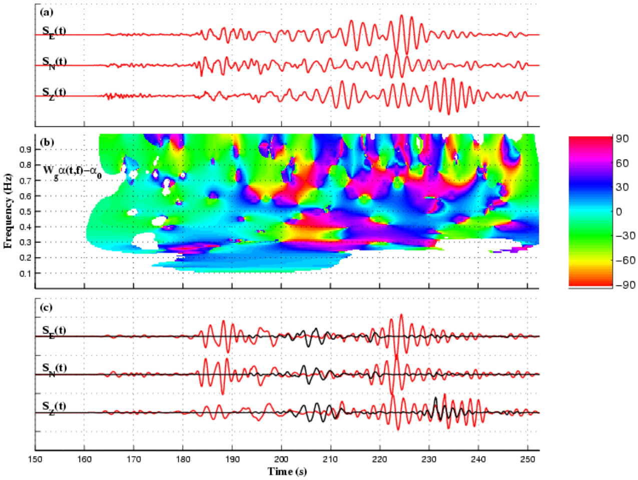
$$A^\pm_{km}(t,f) = \arg \mathcal{W}_g S_k(t,f) \pm \arg \mathcal{W}_g S_m(t,f),$$

$$\Delta t_{km}(t,f) = \frac{4\pi n}{\Omega_k(t,f)+\Omega_m(t,f)}, \quad n \in \mathbb{N}$$

$$\mu_{km} = \Re[\mathcal{W}_g S_k(t,f)]\mathrm{sinc}\left(\frac{\Delta t_{km}(t,f)\Omega_k(t,f)}{2}\right), \quad k,m = x,y,z$$

REAL 3-C DATA FROM A REGIONAL EARTHQUAKE IN THÜRINGEN IN 1989

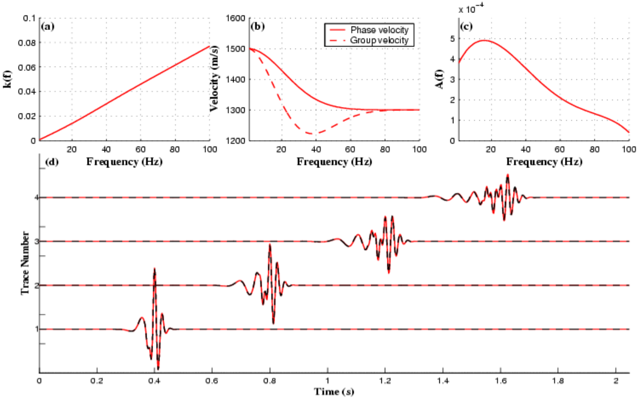
A filtration based on the azimuth of polarization ellipsoid:



ASYMPTOTIC PROPAGATOR IN THE WAVELET SPACE

$$\mathcal{W}_g S_m(t,f) = \mathcal{D}_{\mathcal{W}} \mathcal{W}_g S_k(t,f) = e^{-\alpha(f)D} e^{-i\psi_1(f)} \mathcal{W}_g S_k\left(t - k'(f)D, f\right)$$
$$\psi_1(f) = 2\pi(k(f) - fk'(f))D + 2\pi n$$

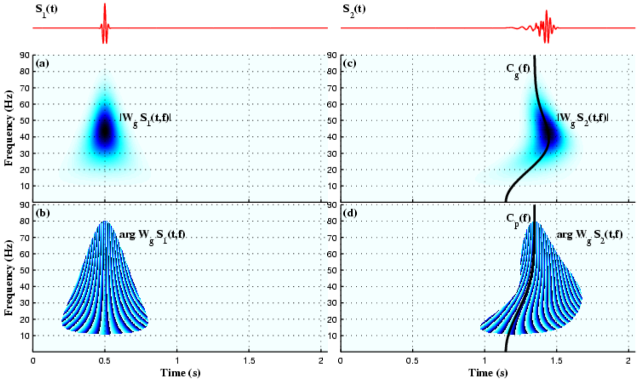
Kulesh at al., *Pure appl. geophys*, **162**, 2005



INTERPRETATION OF ASYMPTOTIC PROPAGATOR

$$\mathcal{W}_g S_m(t,f) = e^{-\alpha(f)D} \cdot \left| \mathcal{W}_g S_k \left(t - \frac{D}{C_g(f)}, f \right) \right| \cdot$$
$$\exp \left[i \arg \mathcal{W}_g S_k \left(t - \frac{D}{C_p(f)} - \frac{n}{f}, f \right) \right]$$

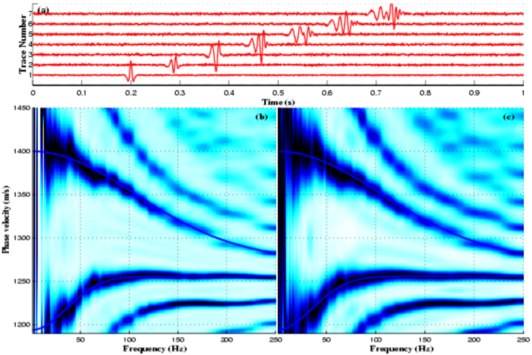
Kulesh at al., *Pure appl. geophys*, (in press), 2007



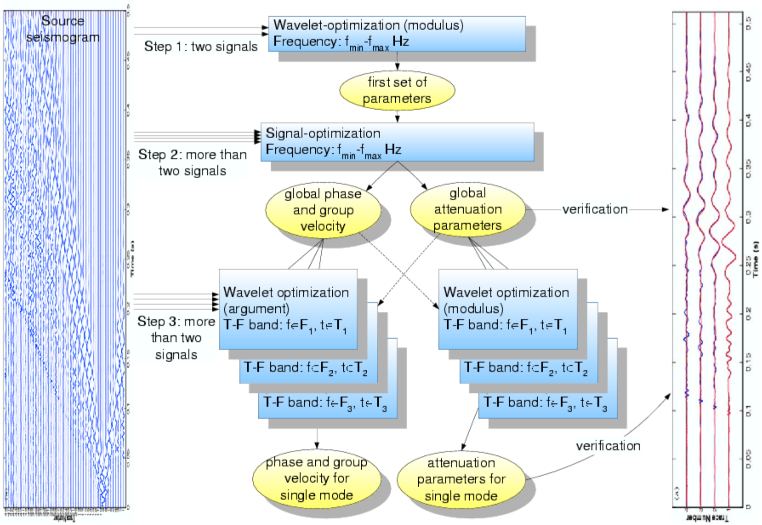
MULTIMODE "FREQUENCY-VELOCITY" ANALYSIS

$$\mathbf{M} = \int_{t_{min}}^{t_{max}} \left| \sum_{k,m} A_k(\tau, f)\, A_m^*\left(\tau - \frac{D_{km}}{c}, f\right) \right| d\tau, \qquad A_k(\tau, f) = \frac{\mathcal{W}_g S_k(\tau,f)}{|\mathcal{W}_g S_k(\tau,f)|}$$

$$\mathbf{M} = \int_{t_{min}}^{t_{max}} \left| \sum_{k,m} e^{iB_k(\tau,f)} e^{-iB_m\left(\tau - \frac{D_{km}}{c}, f\right)} \right| d\tau, \qquad B_k(\tau, f) = \arg \mathcal{W}_g S_k(\tau, f).$$



FLOWCHART OF OPTIMIZATION SEQUENCE

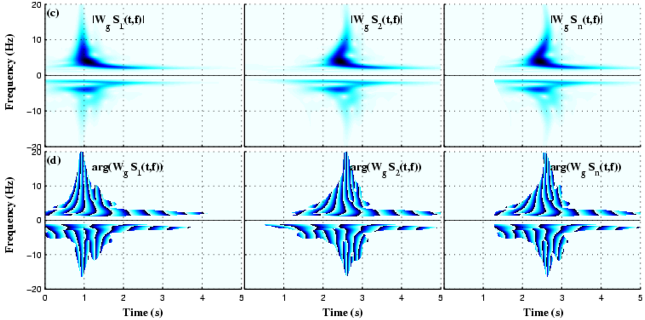


OPTIMIZATION IN THE WAVELET DOMAIN

$$\chi^2(\mathbf{p},\mathbf{q}) = \int\int \left| |\mathcal{W}_g S_2(t,f)| - |\mathcal{D}_\mathcal{W}(\mathbf{p},\mathbf{q})\mathcal{W}_g S_1(t,f)| \right|^2 dt\, df$$

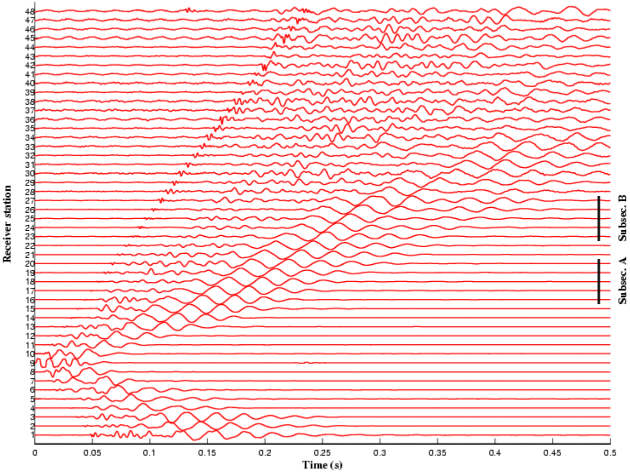
$$\chi^2(\mathbf{p},\mathbf{q}) = \int\int \left| \arg \mathcal{W}_g S_2(t,f) - \arg \mathcal{D}_\mathcal{W}(\mathbf{p},\mathbf{q})\mathcal{W}_g S_1(t,f) \right|^2 dt\, df$$

Holschneider at al., *Geophys. J. Int*, **163**, 2005



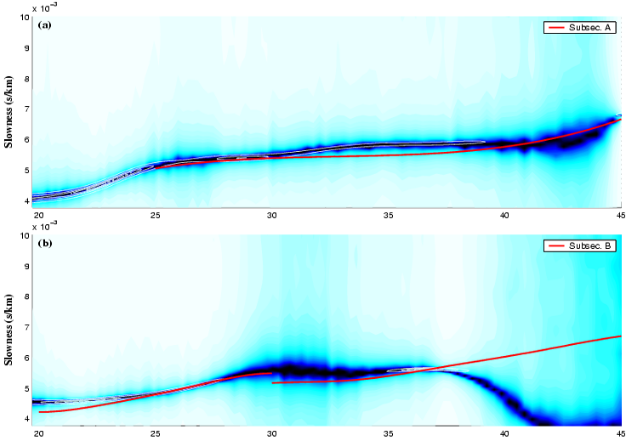
REAL 1-C DATA FROM A SEISMIC EXPERIMENT

A seismogram obtained from a shallow seismic experiment using a sledgehammer as a source



REAL 1-C DATA FROM A SEISMIC EXPERIMENT

Comparison of the slowness estimates from the CWT method (solid curve), Capon's high resolution f-k method (contour lines) and the MUSIC algorithm (background)



CONCLUSIONS

We propose a software package having the following features:

1. object-based implementation of main data types and mathematical objects like Morlet and Cauchy wavelets, some function approximations, 2C and 3C polarization parameters and dispersion parameters;
2. command line and MATLAB interface for transformations like the Fourier transform, CWT and ICWT, 2C and 3C polarization transforms, linear and nonlinear deformations of a wavelet spectrum;
3. command line interface for the optimization in signal and wavelet domains using the algorithm of Levenberg-Marquardt optimization;
4. data import from tabulated and plain ASCII files as well as data export into ASCII files.
5. WWW: http://users.math.uni-potsdam.de/˜gwl