Analysis of wave solutions for the elastic Cosserat medium

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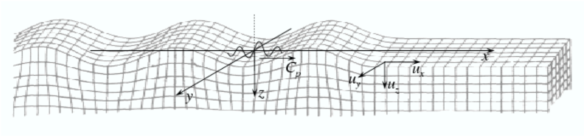
LINEAR WAVE PROPAGATION PROBLEM

- Equation of motion:

$$(\lambda + 2\mu)\,\text{grad}\,\text{div}\,\boldsymbol{u} - \mu\,\text{rot}\,\text{rot}\,\boldsymbol{u} + \boldsymbol{X} = \rho\ddot{\mathbf{u}},$$
$$\boldsymbol{u} = \{u_x(x,z,t), u_y(x,z,t), u_z(x,z,t)\}^T$$

- Problem statement



- Fourier transform of equation (no external loads):
  $(\lambda + 2\mu)\,\text{grad}\,\text{div}\,\hat{\mathbf{u}} - \mu\,\text{rot}\,\text{rot}\,\hat{\mathbf{u}} + \rho\omega^2\hat{\mathbf{u}} = \mathbf{0}$
- Separation of variables:
  $P_{xz}[\hat{\mathbf{u}}(x,z,\omega)] = 0 \quad \rightarrow \quad P_z[\boldsymbol{U}(z), e^{ikx}, k, \omega] = 0$

GENERAL SOLUTION

A monochromatic substitution $\boldsymbol{u} = \boldsymbol{U}(z)e^{i(kx+\omega t)}$ or a non-monochromatic substitution

$$\hat{\boldsymbol{u}} = \boldsymbol{U}(z)e^{ikx}\hat{S}_0(\omega),$$

$$\boldsymbol{u} = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} \boldsymbol{U}(z)e^{i(kx+\omega t)}\hat{S}_0(\omega)\,d\omega$$

give us the general solution:

$$\boldsymbol{U}(z) = \boldsymbol{M}(k,\omega)\cdot\{Ae^{\nu_1 z},\ Be^{-\nu_1 z},\ Ce^{\nu_2 z},\ De^{-\nu_2 z}\}^T,$$

$$\nu_1 = \sqrt{k^2 - \omega^2/C_1^2},\quad \nu_2 = \sqrt{k^2 - \omega^2/C_2^2},$$

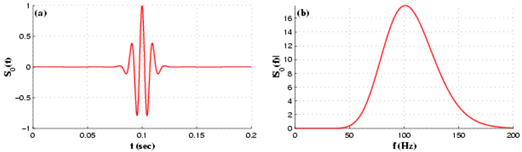
where non-dimensional parameters are used:

$$C_1^2 = \frac{C_l^2}{X_0^2\omega_0^2} = \frac{\lambda+2\mu}{\rho X_0^2\omega_0^2},\qquad C_2^2 = \frac{C_t^2}{X_0^2\omega_0^2} = \frac{\mu}{\rho X_0^2\omega_0^2}$$

CONSTRAINTS

We have three types of constraints:

- Initial conditions are given by the function $\hat{S}_0(\omega)$

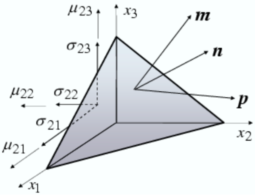


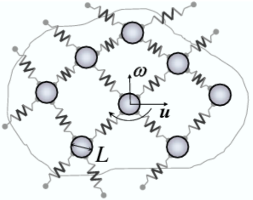
- Boundary conditions are given by restriction on $\boldsymbol{u}$ and (or) $\tilde{\sigma}$ and define a wave type: Body P- and S-waves, Rayleight wave, Lamb wave, Love wave, Stonley wave etc.
- Dispersion equation

$$\det[\mathcal{R}(k,\omega)] = 0 \quad \Longrightarrow \quad k_j = k_j(\omega), \quad j = 1...N$$

HOW CAN BE INTRODUCED THE ROTATION VECTOR IN A LINEAR MODEL?

- Classical elasticity theory ($\boldsymbol{\theta} = 1/2\,\mathrm{rot}\,\boldsymbol{u}$):
  $$(\lambda + 2\mu)\,\mathrm{grad\,div}\,\boldsymbol{u} - \mu\,\mathrm{rot\,rot}\,\boldsymbol{u} + \boldsymbol{X} = \rho\ddot{\boldsymbol{u}}$$
- Reduced Cosserat continuum ($\boldsymbol{\theta}$ and $\boldsymbol{u}$ are dependent):
  $$(\lambda + 2\mu)\,\mathrm{grad\,div}\,\boldsymbol{u} - (\mu + \alpha)\,\mathrm{rot\,rot}\,\boldsymbol{u} + 2\alpha\,\mathrm{rot}\,\boldsymbol{\theta} + \boldsymbol{X} = \rho\ddot{\boldsymbol{u}},$$
  $$2\alpha\,\mathrm{rot}\,\boldsymbol{u} - 4\alpha\boldsymbol{\theta} + \boldsymbol{Y} = j\ddot{\boldsymbol{\theta}}$$
- Cosserat continuum ($\boldsymbol{\theta}$ and $\boldsymbol{u}$ are independent):
  $$(\lambda + 2\mu)\,\mathrm{grad\,div}\,\boldsymbol{u} - (\mu + \alpha)\,\mathrm{rot\,rot}\,\boldsymbol{u} + 2\alpha\,\mathrm{rot}\,\boldsymbol{\theta} + \boldsymbol{X} = \rho\ddot{\boldsymbol{u}},$$
  $$(\beta + 2\gamma)\,\mathrm{grad\,div}\,\boldsymbol{\theta} - (\gamma + \varepsilon)\,\mathrm{rot\,rot}\,\boldsymbol{\theta} + 2\alpha\,\mathrm{rot}\,\boldsymbol{u} - 4\alpha\boldsymbol{\theta} + \boldsymbol{Y} = j\ddot{\boldsymbol{\theta}}$$





LINEAR COSSERAT MODEL

We consider a heterogeneous elastic medium with inclusions as a homogeneous Cosserat continuum, whose point-bodies may rotate and move (no external loads):

$$(\lambda + 2\mu)\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{u} - (\mu + \alpha)\,\mathrm{rot}\,\mathrm{rot}\,\boldsymbol{u} + 2\alpha\,\mathrm{rot}\,\boldsymbol{\theta} = \rho\ddot{\mathbf{u}},$$
$$(\beta + 2\gamma)\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{\theta} - (\gamma + \varepsilon)\,\mathrm{rot}\,\mathrm{rot}\,\boldsymbol{\theta} + 2\alpha\,\mathrm{rot}\,\boldsymbol{u} - 4\alpha\boldsymbol{\theta} = j\ddot{\boldsymbol{\theta}}$$

- $\boldsymbol{p} = \boldsymbol{n} \cdot \tilde{\sigma}$ and $\boldsymbol{m} = \boldsymbol{n} \cdot \tilde{\mu}$ are force and moment of force
- $\boldsymbol{u} = \{u_x, u_y, u_z\}^T$ is displacement vector and $\boldsymbol{\theta} = \{\theta_x, \theta_y, \theta_z\}^T$ is rotation vector
- μ, λ are the Lame constants,
- $\alpha, \beta, \gamma, \varepsilon$ are the physical constants of a material in the framework of the Cosserat medium,
- ρ is the density, j is the moment of inertia density.

LINEAR COSSERAT MODEL

- Asymmetric strain tensor
  $\tilde{\gamma} = \nabla \boldsymbol{u} - \tilde{\boldsymbol{E}} \cdot \boldsymbol{\theta}$
- Asymmetric torsion bending tensor
  $\tilde{\chi} = \nabla \boldsymbol{\theta}$
- Asymmetric stress tensor
  $\tilde{\sigma} = 2\mu\tilde{\gamma}^{(S)} + 2\alpha\tilde{\gamma}^{(A)} + \lambda I_1(\tilde{\gamma})\tilde{e}$
- Asymmetric couple-stress tensor
  $\tilde{\mu} = 2\gamma\tilde{\chi}^{(S)} + 2\varepsilon\tilde{\chi}^{(A)} + \beta I_1(\tilde{\chi})\tilde{e}$
- Equilibrium equations
  $\nabla \cdot \tilde{\sigma} + \boldsymbol{X} = \rho\ddot{\boldsymbol{u}},$
  $\tilde{\sigma}^T : \tilde{\boldsymbol{E}} + \nabla \cdot \tilde{\mu} + \boldsymbol{Y} = j\ddot{\boldsymbol{\theta}}$
- $(.)^{(S)}$ and $(.)^{(A)}$ denote the symmetric and antisymmetric parts of tensor
- $\tilde{\boldsymbol{E}}$ is the Levi-Civita tensor of the third rank

GENERAL SOLUTION

Solution steps:

- Fourier transform of equation:

$$(\lambda + 2\mu)\text{grad div}\,\hat{\mathbf{u}} - (\mu + \alpha)\text{rot rot}\,\hat{\mathbf{u}} + 2\alpha\,\text{rot}\,\hat{\boldsymbol{\theta}} + \rho\omega^2\hat{\mathbf{u}} = \mathbf{0},$$
$$(\beta + 2\gamma)\text{grad div}\,\hat{\boldsymbol{\theta}} - (\gamma + \varepsilon)\text{rot rot}\,\hat{\boldsymbol{\theta}} + 2\alpha\,\text{rot}\,\hat{\mathbf{u}} - (4\alpha - j\omega^2)\hat{\boldsymbol{\theta}} = \mathbf{0}$$

- Non-monochromatic substitution:

$$\hat{\mathbf{u}} = \{U_x(z),\ U_y(z),\ U_z(z)\}^T e^{ikx}\hat{S}_0(\omega),$$
$$\hat{\boldsymbol{\theta}} = \{W_x(z),\ W_y(z),\ W_z(z)\}^T e^{ikx}\hat{S}_0(\omega)$$

- Solution form

$$\mathbf{u}(x, z, k(\omega), t) = \int_{-\infty}^{\infty} \mathbf{U}(z)e^{i(kx+\omega t)}\hat{S}_0(\omega)\,d\omega,$$

$$\boldsymbol{\theta}(x, z, k(\omega), t) = \int_{-\infty}^{\infty} \mathbf{W}(z)e^{i(kx+\omega t)}\hat{S}_0(\omega)\,d\omega$$

EQUATIONS TO BE SOLVED

- Longitudinal mode $U_x(z),\ U_z(z),\ W_y(z)$

$$(\mu+\alpha)U_x''(z)+(\rho\omega^2-k^2(\lambda+2\mu))U_x(z)+$$
$$+ik(\lambda+\mu-\alpha)U_z'(z)-2\alpha W_y'(z)=0,$$
$$(\lambda+2\mu)U_z''(z)+(\rho\omega^2-k^2(\mu+\alpha))U_z(z)+$$
$$+ik(\lambda+\mu-\alpha)U_x'(z)+2ik\alpha W_y(z)=0,$$
$$(\gamma+\varepsilon)W_y''(z)+(j\omega^2-k^2(\gamma+\varepsilon)-4\alpha)W_y(z)+$$
$$+2\alpha U_x'(z)-2ik\alpha U_z(z)=0$$

- Transverse mode $U_y(z),\ W_x(z),\ W_z(z)$

$$(\gamma+\varepsilon)W_x''(z)+(j\omega^2-k^2(\beta+2\gamma)-4\alpha)W_x(z)+$$
$$+ik(\beta+\gamma-\varepsilon)W_z'(z)-2\alpha U_y'(z)=0,$$
$$(\beta+2\gamma)W_z''(z)+(j\omega^2-k^2(\gamma+\varepsilon)-4\alpha)W_z(z)+$$
$$+ik(\beta+\gamma-\varepsilon)W_x'(z)+2ik\alpha U_y(z)=0,$$
$$(\mu+\alpha)U_y''(z)+(\rho\omega^2-k^2(\mu+\alpha))U_y(z)+$$
$$+2\alpha W_x'(z)-2ik\alpha W_z(z)=0.$$

LONGITUDINAL P-WAVES

- Representation of P-wave

$$U_x(z) = U_x, U_y(z) = 0, U_z(z) = 0,$$
$$W_x(z) = W_x, W_y(z) = 0, W_z(z) = 0$$

- Two disperion equations:

$$(\rho\omega^2 - k^2(\lambda + 2\mu))U_x(z) = 0,$$
$$(j\omega^2 - k^2(\beta + 2\gamma) - 4\alpha)W_x(z) = 0$$

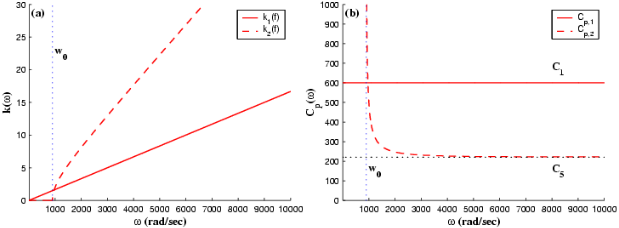
- Two disperion curves:

$$k_1(\omega) = \omega\sqrt{\frac{\rho}{(\lambda+2\mu)}},$$
$$k_2(\omega) = \sqrt{\frac{j\omega^2}{(\beta+2\gamma)} - \frac{4\alpha}{(\beta+2\gamma)}}$$

NON-DIMENSIONAL SOLUTION

$$k_1(\omega) = \frac{\omega}{C_1}, \quad k_2(\omega) = \sqrt{\frac{\omega^2}{C_5^2} - k_0^2},$$

$$C_1^2 = \frac{\lambda+2\mu}{\rho X_0^2 \omega_0^2}, \quad C_5^2 = \frac{\beta+2\gamma}{j X_0^2 \omega_0^2}, \quad w_0 = 2\sqrt{\frac{\alpha}{j}}, \quad C_{p,i} = \frac{\omega}{k_i(\omega)}$$



New parameter C_5 describes the boundary velocity of longitudinal rotation wave

TRANSVERSAL S-WAVES

- Representation of S-wave

$$U_x(z) = 0, U_y(z) = U_y, U_z(z) = U_z,$$
$$W_x(z) = 0, W_y(z) = W_y, W_z(z) = W_z$$

- Disperion equations:

$$(\gamma+\varepsilon)(\mu+\alpha)k^4 + (4\alpha\mu - (j(\mu+\alpha) + \rho(\gamma+\varepsilon))\omega^2)k^2 + \\ + j\rho\omega^4 - 4\alpha\rho\omega^2 = 0$$

- Non-dimensional form:

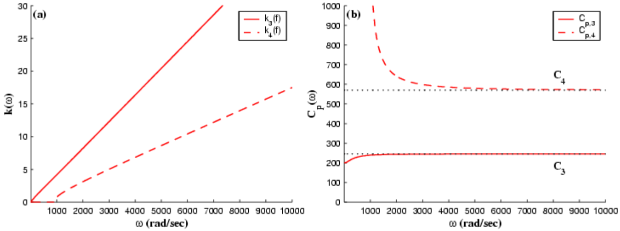
$$k^4 + \left(4A^2 - \frac{C_3^2+C_4^2}{C_3^2C_4^2}\omega^2\right)k^2 + \frac{\omega^4}{C_3^2C_4^2} - \frac{4A^2}{C_2^2}\omega^2 = 0,$$
$$A^2 = X_0^2\frac{\mu\alpha}{(\mu+\alpha)(\gamma+\varepsilon)}, \quad C_2^2 = \frac{\mu}{\rho X_0^2\omega_0^2}, \quad C_3^2 = \frac{\mu+\alpha}{\rho X_0^2\omega_0^2}, \quad C_4^2 = \frac{\gamma+\varepsilon}{jX_0^2\omega_0^2}$$

NON-DIMENSIONAL SOLUTION

$$k_3(\omega) = \sqrt{A_p(\omega)}, \quad k_4(\omega) = \sqrt{A_m(\omega)}, \quad \text{where}$$

$$A_{p,m}(\omega) = \frac{C_4^2 + C_3^2}{2 C_3^2 C_4^2}\omega^2 - 2A^2 \pm \sqrt{\mathcal{D}(\omega)},$$

$$\mathcal{D}(\omega) = \omega^4 \frac{C_3^4 + C_4^4 - 2 C_4^2 C_3^2}{4 C_3^4 C_4^4} - 2\omega^2 \frac{A^2 (C_3^2 C_2^2 + C_4^2 C_2^2 - 2 C_4^2 C_3^2)}{C_3^2 C_4^2 C_2^2} + 4A^4$$



GENERAL SOLUTION FOR LONGITUDINAL WAVES

$$u_x(x,z,t) = \int_{-\infty}^{\infty} \Big\{ D_1 ike^{-\nu_1 z} + D_2\nu_2 e^{-\nu_2 z} + D_3\nu_3 e^{-\nu_3 z} + D_4 ike^{\nu_1 z} - D_5\nu_2 e^{\nu_2 z} - D_6\nu_3 e^{\nu_3 z} \Big\} e^{i(kx+\omega t)} \hat{S}_0(\omega)\, d\omega,$$

$$u_z(x,z,t) = \int_{-\infty}^{\infty} \Big\{ -D_1\nu_1 e^{-\nu_1 z} + D_2 ike^{-\nu_2 z} + D_3 ike^{-\nu_3 z} + D_4\nu_1 e^{\nu_1 z} + D_5 ike^{\nu_2 z} + D_6 ike^{\nu_3 z} \Big\} e^{i(kx+\omega t)} \hat{S}_0(\omega)\, d\omega,$$

$$\theta_y(x,z,t) = \frac{B}{2} \int_{-\infty}^{\infty} \Big\{ D_2 \Big(A_m - \frac{\omega^2}{C_3^2}\Big) e^{-\nu_2 z} + D_3 \Big(A_p - \frac{\omega^2}{C_3^2}\Big) e^{-\nu_3 z} + D_5 \Big(A_m - \frac{\omega^2}{C_3^2}\Big) e^{\nu_2 z} + D_6 \Big(A_p - \frac{\omega^2}{C_3^2}\Big) e^{\nu_3 z} \Big\} e^{i(kx+\omega t)} \hat{S}_0(\omega)\, d\omega,$$

where $\nu_1 = \sqrt{k^2 - \frac{\omega^2}{C_1^2}}$, $\nu_{2,3} = \sqrt{k^2 - A_{m,p}}$, $B = \frac{\alpha+\mu}{\alpha}$

GENERAL SOLUTION FOR TRANSVERSAL WAVES

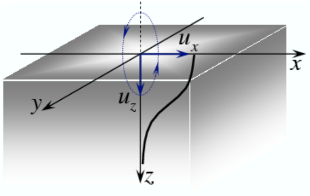
$$u_y(x,z,t) = \frac{F}{2}\int_{-\infty}^{\infty}\Big\{\Big(A_m - \frac{\omega^2}{C_4^2} + \frac{4}{F}\Big)\Big(E_2 e^{-\xi_2 z} + E_5 e^{\xi_2 z}\Big) + \Big(A_p - \frac{\omega^2}{C_4^2} + \frac{4}{F}\Big)\Big(E_3 e^{-\xi_3 z} + E_6 e^{\xi_3 z}\Big)\Big\} e^{i(kx+\omega t)}\hat{S}_0(\omega)\,d\omega,$$

$$\theta_x(x,z,t) = \int_{-\infty}^{\infty}\Big\{E_1 ike^{-\xi_1 z} + E_2\xi_2 e^{-\xi_2 z} + E_3\xi_3 e^{-\xi_3 z} + E_4 ike^{\xi_1 z} - E_5\xi_2 e^{\xi_2 z} - E_6\xi_3 e^{\xi_3 z}\Big\} e^{i(kx+\omega t)}\hat{S}_0(\omega)\,d\omega,$$

$$\theta_z(x,z,t) = \int_{-\infty}^{\infty}\Big\{-E_1\xi_1 e^{-\xi_1 z} + E_2 ike^{-\xi_2 z} + E_3 ike^{-\xi_3 z} + E_4\xi_1 e^{\xi_1 z} + E_5 ike^{\xi_2 z} + E_6 ike^{\xi_3 z}\Big\} e^{i(kx+\omega t)}\hat{S}_0(\omega)\,d\omega,$$

where $\xi_1 = \sqrt{k^2 - \frac{\omega^2}{C_5^2} + \frac{4C_4^2}{FC_5^2}}$, $\xi_{2,3} = \sqrt{k^2 - A_{m,p}}$, $F = \frac{\gamma+\varepsilon}{\alpha X_0^2}$

RAYLEIGH WAVE



- Amplitude of displacement components fades along the z-axis: $D_4 = D_5 = D_6 = 0$
- The boundary conditions at the surface $z = 0$ require that normal forces and moments be zero and in the dimensionless form are

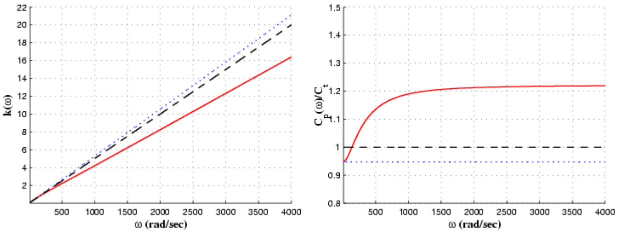
$$\sigma_{zx}|_{z=0} = 0, \quad \sigma_{zy}|_{z=0} = 0, \quad \sigma_{zz}|_{z=0} = 0$$
$$\mu_{zx}|_{z=0} = 0, \quad \mu_{zy}|_{z=0} = 0, \quad \mu_{zz}|_{z=0} = 0$$

RAYLEIGH WAVE

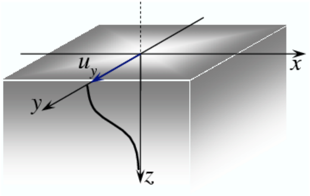
Dispersion equation

$$\det(\mathcal{R}_r(\nu_1, \nu_2, \nu_3)) = 0,$$

$$\mathcal{R}_r(p_1, p_2, p_3) = \begin{bmatrix} 2k^2 - \frac{\omega^2}{C_2^2} & -2ikp_2 & -2ikp_3 \\ 2ikp_1 & 2k^2 - \frac{\omega^2}{C_2^2} & 2k^2 - \frac{\omega^2}{C_2^2} \\ 0 & p_2\left(A_m - \frac{\omega^2}{C_3^2}\right) & p_3\left(A_p - \frac{\omega^2}{C_3^2}\right) \end{bmatrix}$$



SURFACE TRANSVERSE WAVE



- Amplitude of displacement components fades along the z-axis: $E_4 = E_5 = E_6 = 0$
- The boundary conditions at the surface $z = 0$ require that normal forces and moments be zero and in the dimensionless form are

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$$\mu_{zx}\big|_{z=0} = 0, \quad \mu_{zy}\big|_{z=0} = 0, \quad \mu_{zz}\big|_{z=0} = 0$$

SURFACE TRANSVERSE WAVE

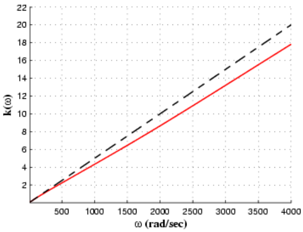
Dispersion equation

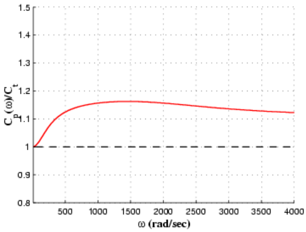
$$\det(\mathcal{R}_t(\xi_1, \xi_2, \xi_3)) = 0,$$

$$\mathcal{R}_t(p_1, p_2, p_3) =$$

$$\begin{bmatrix} \frac{2ik}{1-B} & p_2\left(2 + \frac{A_m C_4^2 - \omega^2}{2A^2 C_4^2}\right) & p_3\left(2 + \frac{A_p C_4^2 - \omega^2}{2A^2 C_4^2}\right) \\ ikp_1(1+C) & p_2^2 + k^2 C & p_3^2 + k^2 C \\ \left(\frac{C_5^2}{C_4^2} - C - 1\right)k^2 - p_1^2 \frac{C_5^2}{C_4^2} & ikp_2(1+C) & ikp_3(1+C) \end{bmatrix}$$

$$C = \frac{\gamma - \varepsilon}{\gamma + \varepsilon}$$





SURFACE TRANSVERSE WAVE

Displacement and rotation vectors components

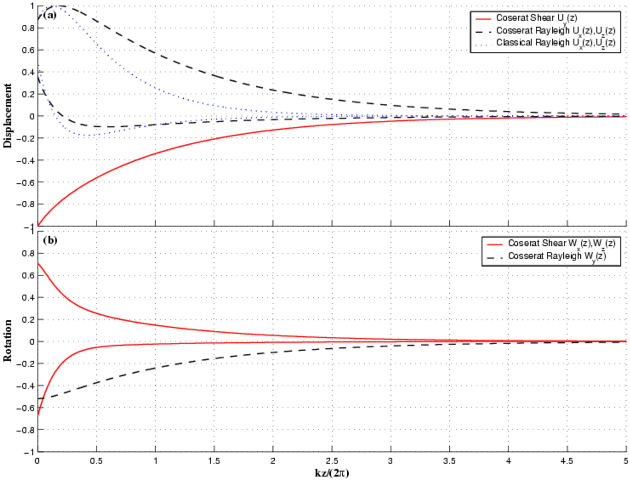
$$\theta_x(x,z,t) = \int\limits_{-\infty}^{\infty} \left\{ k e^{-\xi_1 z} + \frac{G_1}{k} e^{-\xi_2 z} - \frac{G_2}{k} e^{-\xi_3 z} \right\} e^{i(kx+\omega t-\pi/2)} \hat{S}_0(\omega)\, d\omega,$$

$$u_y(x,z,t) = \frac{B-1}{2A^2B} \int\limits_{-\infty}^{\infty} \left\{ \frac{G_1}{k\xi_2} \left(A_m - \frac{\omega^2}{C_4^2} + \frac{4A^2B}{B-1} \right) e^{-\xi_2 z} - \frac{G_2}{k\xi_3} \left(A_p - \frac{\omega^2}{C_4^2} + \frac{4A^2B}{B-1} \right) e^{-\xi_3 z} \right\} e^{i(kx+\omega t-\pi/2)} \hat{S}_0(\omega)\, d\omega,$$

$$\theta_z(x,z,t) = \int\limits_{-\infty}^{\infty} \left\{ \xi_1 e^{-\xi_1 z} + \frac{G_1}{\xi_2} e^{-\xi_2 z} - \frac{G_2}{\xi_3} e^{-\xi_3 z} \right\} e^{i(kx+\omega t)} \hat{S}_0(\omega)\, d\omega$$

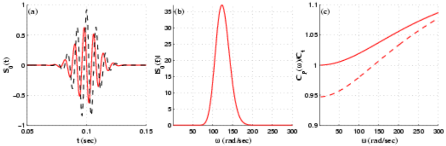
Note that in addition to the elliptic surface Rayleigh wave, it is also possible to observe another wave type, a surface wave whose one component is parallel to the boundary surface and perpendicular to the propagation direction.

SURFACE TRANSVERSE WAVE

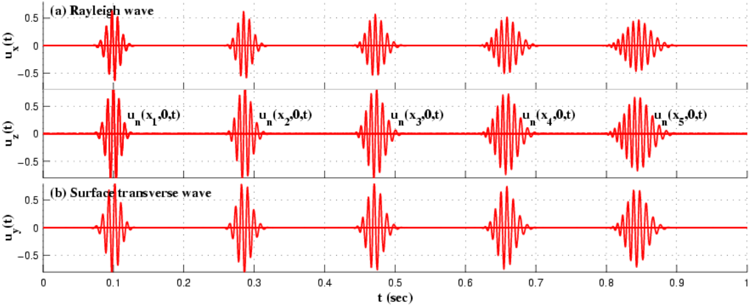


AN EXAMPLE - SURFACE SEISMOGRAM

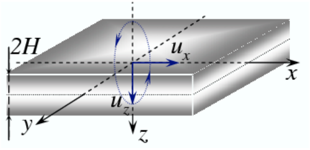
Source signal S_0 and dispersion properties of the medium:



Propagated waveform:



LAMB WAVE



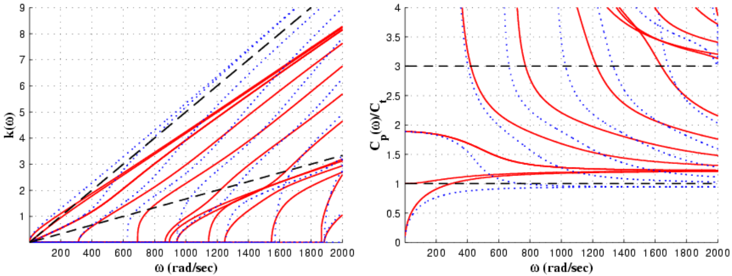
- Let us consider a free loaded plate with the thickness $2H$, characteristic length $X_0 = H$
- The boundary conditions at the surfaces $z = \pm 1$ require that normal forces and moments are zero and in the dimensionless form are

$$\sigma_{zx}\big|_{z=\pm 1} = 0, \quad \sigma_{zy}\big|_{z=\pm 1} = 0, \quad \sigma_{zz}\big|_{z=\pm 1} = 0$$
$$\mu_{zx}\big|_{z=\pm 1} = 0, \quad \mu_{zy}\big|_{z=\pm 1} = 0, \quad \mu_{zz}\big|_{z=\pm 1} = 0$$

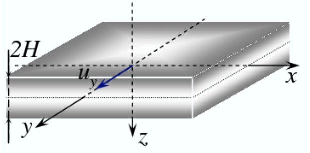
LAMB WAVE

Lamb wave with components u_x, u_z and θ_y is described by the dispersion equation

$$\det \begin{bmatrix} \mathcal{R}_r(\nu_1,\nu_2,\nu_3)\mathrm{diag}(e^{-\nu_n}) & \mathcal{R}_r(-\nu_1,-\nu_2,-\nu_3)\mathrm{diag}(e^{\nu_n}) \\ \mathcal{R}_r(\nu_1,\nu_2,\nu_3)\mathrm{diag}(e^{\nu_n}) & \mathcal{R}_r(-\nu_1,-\nu_2,-\nu_3)\mathrm{diag}(e^{-\nu_n}) \end{bmatrix} = 0$$



TRANSVERSE LAMB WAVE



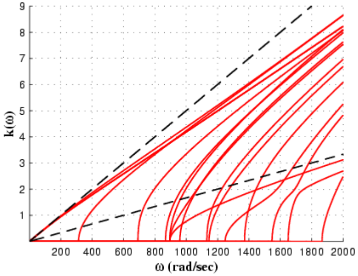
- Let us consider a free loaded plate with the thickness $2H$, characteristic length $X_0 = H$
- The boundary conditions at the surfaces $z = \pm 1$ require that normal forces and moments are zero and in the dimensionless form are

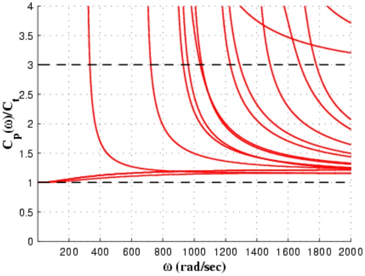
$$\sigma_{zx}\big|_{z=\pm 1} = 0, \quad \sigma_{zy}\big|_{z=\pm 1} = 0, \quad \sigma_{zz}\big|_{z=\pm 1} = 0$$
$$\mu_{zx}\big|_{z=\pm 1} = 0, \quad \mu_{zy}\big|_{z=\pm 1} = 0, \quad \mu_{zz}\big|_{z=\pm 1} = 0$$

TRANSVERSE LAMB WAVE

Transverse wave with components u_y, θ_x and θ_z has the following dispersion equation:

$$\det\begin{bmatrix} \mathcal{R}_t(\xi_1,\xi_2,\xi_3)\mathrm{diag}(e^{-\xi_n}) & \mathcal{R}_t(-\xi_1,-\xi_2,-\xi_3)\mathrm{diag}(e^{\xi_n}) \\ \mathcal{R}_t(\xi_1,\xi_2,\xi_3)\mathrm{diag}(e^{\xi_n}) & \mathcal{R}_t(-\xi_1,-\xi_2,-\xi_3)\mathrm{diag}(e^{-\xi_n}) \end{bmatrix} = 0$$





CONCLUSION

- In this study we have obtained some analytical solutions corresponding to P- and S-waves, Rayleigh wave and surface transverse wave in a half-space as well as Lamb wave and transverse wave in a thin layer within the framework of linear Cosserat model.
- These solutions can be divided into two groups, one of which corresponds to the well-investigated elliptical wave and the other — to the transverse wave with depth-dependent decay which does not have any analogy in the classical theory of elasticity.
- We have compared the solution for all observed waves to classical case with help of numerical illustrations.

OPEN QUESTIONS

- Analysis of displacemet components for the Lamb and transverse waves in a thin layer.
- Construction and analysis of analytical solution for cylindrical waves with two characteristic sizes.
- Collection of experimental results for the wave propagation problem in medium with microstructure.
- In which wave propagation problems we can use the linear Cosserat model and when do we need to use a non-linear approach?
- The main result of the present work is the theoretical existence of surface transversal waves. Can we construct an experiment to demonstrate it?