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Instantaneous Polarization Attributes Based on Adaptive Covariance Method

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Instantaneous Polarization Attributes Based on Adaptive Covariance Method

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SUMMARY

We propose a new approach of polarization analysis for multicomponent signals which is based on an adaptive method of selecting the time window to be used for the covariance matrix. The advantage of our approach is that we obtain all the polarization parameters at each point in time (no interpolation is required) and that the length of the time window for the covariance matrix computation is adaptively adjusted to the smallest period of the signal component in the time window. Therefore we are relieved from the constraint of selecting the window length around the arrival of interest which is critical to the success of the polarization analysis. We show some numerical examples for illustration.

Key words: polarization attributes, covariance method, multicomponent signal

1 INTRODUCTION

A triaxial sensor yields three traces $S_x(t)$, $S_y(t)$ and $S_z(t)$. The polarization analysis is usually done by the eigenanalysis of the cross-energy matrix M (Flinn, 1965; Montalbetti

& Kanasewich, 1970; Esmersoy, 1984; Jurkevics, 1988)

$$M(\xi) = \begin{bmatrix} I_x(\xi) & J_{xy}(\xi) & J_{xz}(\xi) \\ J_{xy}(\xi) & I_y(\xi) & J_{yz}(\xi) \\ J_{xz}(\xi) & J_{yz}(\xi) & I_z(\xi) \end{bmatrix}, \tag{1}$$

where

$$I_k(\xi) = \frac{1}{T} \int\limits_{\xi-T/2}^{\xi+T/2} (S_k(\tau) - \mu_k(\xi))^2 d\tau,$$

$$J_{km}(\xi) = \frac{1}{T} \int\limits_{\xi-T/2}^{\xi+T/2} (S_k(\tau) - \mu_k(\xi))(S_m(\tau) - \mu_m(\xi)) d\tau, \qquad k, m = (x, y, z).$$

In the above equations ξ is the position of the center of the time window T, around which the covariance matrix is computed, and $\mu_k(\xi)$ is the mean value of each signal component in the analysis window. Eigenanalysis performed on the matrix $M(\xi)$ yields the principal component decomposition of the energy for the considered time window. Such a decomposition yields three eigenvalues $\lambda_k(\xi)$ and three corresponding eigenvectors $\mathbf{v}_k(\xi)$ that fully characterize the magnitudes and directions of the principal component of the ellipsoid that approximates the particle motion in the considered time window. The eigenvalues and eigenvectors can be used to design filters for noise reduction, for separation between elliptically and linearly polarized events (Montalbetti & Kanasewich, 1970; Jurkevics, 1988; Jackson et al., 1991; Reading et al., 2001).

One critical issue in the application of the covariance method is the selection of the window size. Moreover, since the eigenvalues (eigenvectors) are estimated once for each window, one needs an interpolation procedure to determine these parameters at each sample point prior to using them for filtering or denoising purposes. Alternative methods have been proposed to overcome these constraints by (Vidale, 1986) who used the Hilbert transform of the multicomponent signal (component-wise) to construct the covariance matrix and subsequently determine the instantaneous ellipticity. However with that method, one still needs to specify the length of time window for the estimation of the polarization parameters. (Morozov & Smithson, 1996) proposed another approach that allows an estimate of all the polarization parameters at each time point and for any number of components but which is limited to the case of an ellipse in 3D space. Therefore with the latter method it is not possible to assess the strength of the polarization direction that is associated to the third eigenvalue obtained from equation (1).

In this contribution we propose an approach to overcome the constraint related to the window length by proposing and adaptive method for its selection. With the proposed method it is possible to characterize the instantaneous polarization ellipsoid.

2 ADAPTIVE CROSS-ENERGY MATRIX METHOD

Let $S_k^H(t)$ be the Hilbert transform of the signal $S_k(t)$, $k = (x, y, z)$. The corresponding analytic signals $C_k(t)$ are

$$C_k(t) = S_k(t) + iS_k^H(t).$$

Locally around t we may write with the help of $C_k(t)$

$$S_k(t+\tau) \simeq \frac{1}{2}\left(C_k(t)e^{i\Omega_k(t)\tau} + C_k^*(t)e^{-i\Omega_k(t)\tau}\right) = |C_k(t)|\cos\left(\Omega_k(t)\tau + \arg C_k(t)\right), \tag{2}$$

where $\Omega_k(t) = \frac{d}{dt}\arg C_k(t)$ are the instantaneous frequencies and $(\cdot)^*$ indicates the complex conjugate.

Using the approximations (2) for each component $S_k(t)$ in building the cross-energy matrix yields

$$\begin{aligned}
\tilde{M}(t) &= \begin{bmatrix} \tilde{I}_{xx}(t) & \tilde{I}_{xy}(t) & \tilde{I}_{xz}(t) \\ \tilde{I}_{xy}(t) & \tilde{I}_{yy}(t) & \tilde{I}_{yz}(t) \\ \tilde{I}_{xz}(t) & \tilde{I}_{yz}(t) & \tilde{I}_{zz}(t) \end{bmatrix}, \\
\tilde{I}_{km}(t) &= \tfrac{1}{T_{km}(t)} \int\limits_{-T_{km}(t)/2}^{T_{km}(t)/2} (S_k(t+\tau) - \mu_k)(S_m(t+\tau) - \mu_m)d\tau = \\
&= |C_k(t)||C_m(t)| \cdot \Big\{\mathrm{sinc}\left[\tfrac{\Omega_k(t)-\Omega_m(t)}{2}T_{km}(t)\right]\cos\left[\arg C_k(t) - \arg C_m(t)\right] + \\
&\quad \mathrm{sinc}\left[\tfrac{\Omega_k(t)+\Omega_m(t)}{2}T_{km}(t)\right]\cos\left[\arg C_k(t) + \arg C_m(t)\right]\Big\} - \mu_k\mu_m,
\end{aligned} \tag{3}$$

respectively, where mean value μ_k is defined as

$$\mu_k = \frac{1}{T_{km}(t)} \int\limits_{-T_{km}(t)/2}^{T_{km}(t)/2} S_k(t+\tau)d\tau = \Re[C_k(t)]\mathrm{sinc}[T_{km}(t)\Omega_k(t)/2].$$

Sine cardinal function sinc(x) is defined as

$$\operatorname{sinc}(x) = \begin{cases} 1 & \text{for } x = 0, \\ \frac{sin(x)}{x} & \text{otherwise} \end{cases}$$

and $\Re(\cdot)$ is a real part of complex value.

$T_{km}(t)$ represents the adaptive time window which is determined from the instantaneous frequencies at time t as defined below. Note that unlike $M(\xi)$, the matrix $\tilde{M}(t)$ is defined for each time t. Therefore the eigenanalysis of $\tilde{M}(t)$ will give the instantaneous polarization attributes. This constitutes the main advantage over the eigenanalysis of $M(\xi)$ in equation (1), where the polarization attributes are estimated at only one time location within each time window. In $\tilde{M}(t)$ it is possible to choose the length of adaptive time window to be the same for all the elements as

$$T_{km}(t) = \frac{6\pi N}{\Omega_x(t) + \Omega_y(t) + \Omega_z(t)}. \tag{4}$$

Alternatively one can also define a window length that is specific to each element of the covariance matrix (3) as

$$T_{km}(t) = \frac{4\pi N}{\Omega_k(t) + \Omega_m(t)}, \tag{5}$$

which yields more accurate estimates for the polarization attributes. In equations (4), (5) N is integer and must be greater or equal to 1 for an accurate estimate of the instantaneous polarization parameters. Higher value of N makes it possible to determine the instantaneous attributes of the polarization ellipsoid at the cost of less accurate approximation of $\tilde{S}_k(\tau)$ using equation (2).

Given the three eigenvalues $\lambda_1(t) \geq \lambda_2(t) \geq \lambda_3(t)$ and the corresponding eigenvectors $\mathbf{v}_k(t)$, obtained from the eigenanalysis of $\tilde{M}(t)$, one can compute the usual instantaneous attributes for the polarization ellipsoid:

$$\begin{gathered} R_{max}(t) = \sqrt{\lambda_1(t)}, \quad R_{med}(t) = \sqrt{\lambda_2(t)}, \quad R_{min}(t) = \sqrt{\lambda_3(t)}, \\ \mathbf{R}_{max}(t) = R_{max}(t)\tfrac{\mathbf{v}_1(t)}{\|\mathbf{v}_1(t)\|}, \quad \mathbf{R}_{med}(t) = R_{med}(t)\tfrac{\mathbf{v}_2(t)}{\|\mathbf{v}_2(t)\|}, \quad \mathbf{R}_{min}(t) = R_{min}(t)\tfrac{\mathbf{v}_3(t)}{\|\mathbf{v}_3(t)\|}, \\ \theta_0(t) = \arctan\left[\tfrac{v_{1y}(t)}{v_{1x}(t)}\right], \quad \delta_0(t) = \arctan\left[\tfrac{v_{1z}(t)}{\sqrt{v_{1x}(t)^2 + v_{1y}(t)^2}}\right], \end{gathered} \tag{6}$$

where $R_{max}(t)$ is the instantaneous major axis, $R_{med}(t)$ and $R_{min}(t)$ are the two instantaneous minor axes, $\theta_0(t)$ and $\delta_0(t)$ are the strike and the dip of the major direction of polarization respectively. Bold characters indicate that the considered parameter is vectorial rather than scalar.

The smallest eigenvector $\mathbf{v}_3(t)$ is parallel to the planarity vector $\mathbf{v}_p(t)$ defined as (Schimmel & Gallart, 2003) $\mathbf{v}_p(t) = \mathbf{R}_{max}(t) \times \mathbf{R}_{med}(t)$ from the major and minor axis as obtained using Morozov's method (Morozov & Smithson, 1996). Its direction cosines $\theta_x(t)$, $\theta_y(t)$, $\theta_z(t)$ can be used to determine the plane containing the major ellipse in 3D space:

$$\theta_k(t) = \arccos\left[\frac{v_{3k}(t)}{\|\mathbf{v}_3(t)\|}\right], \quad k = (x, y, z). \tag{7}$$

3 NUMERICAL EXAMPLES

To illustrate the performance of the proposed method we compare the polarization attributes to those obtained with other methods. The analysis is performed on a synthetic three-component record containing arrivals with different types of polarization. Figure (1) shows the synthetic three-component record. Event A simulates a stationary ellipse, event B simulates a rotating ellipse, and event C is that of a linearly polarized event. Event D is a stationary ellipse in 3D space, and event E corresponds to that of a rotating ellipsoid. The first three events are contained in the x-y plane while the last two are felt across all directions.

In Figure (2a) the major and minors axes of polarization computed from the proposed method are plotted next to those obtained from the standard covariance method and Morozov's method (Morozov & Smithson, 1996). First we note that the new method and that of Morozov give similar results for the major axis and the largest minor axis for all the events. However with the standard covariance method the determination of these axes is less accurate as evidenced by the oscillating curves in Figure (2a). This is due to the constraint of finding the optimal time window as mentioned earlier. Note that for the standard covariance method the time window was shifted by one time sample so that the polarization parameters are determined for each time expect for some points at the end of the time series.

For the ellipsoidal event E we note that (aside from the oscillations observed for the curves from the standard covariance method) all methods give similar results for the major and the largest minor axes estimates but only the standard covariance method and the new method can provide an estimate for the third (smallest) polarization axis. However the estimate of the third polarization is not unique and is determined by the selected window length for the standard covariance method and by the value of the integer N in equations (5) as explained earlier.

Figures (2b-d) show plots of other polarization attributes from the different methods for comparison. In Figure (2b) the ellipticity ratio is shown. For the events A, B, C, D, the ratio $R_{med}(t)/R_{max}(t)$ is plotted; for the last event both $R_{med}(t)/R_{max}(t)$ and $R_{min}(t)/R_{max}(t)$ are plotted as well. Consistent results are observed for all methods. With these parameters one can clearly distinguish the linearly polarized events from those with elliptical or ellipsoidal polarization. However with the standard covariance method the estimated attributes show a high degree of variance.

Figure (2c) shows the direction cosines of third eigenvector $\mathbf{v}_3(t)$ from the proposed method (7) and the standard method plotted next to the direction cosines of the $v_p(t)$ defined from Morozov's major and minor axes. Here also the estimates are consistent across all methods but with high variance of the estimates from the standard method as observed for the other attributes.

Figure (2d) depicts the instantaneous frequencies $\Omega_x(t)$, $\Omega_y(t)$, $\Omega_z(t)$ from the signals $S_x(t)$, $S_y(t)$, $S_z(t)$ in Figure (1) respectively. The adaptive method for choosing the window length is based on these parameters as can be seen from equations (4), (5). These parameters can also be used to characterize the dynamics within each polarization ellipse or ellipsoid along with the geometrical parameters discussed above.

4 CONCLUSIONS

We have introduced a method for instantaneous polarization attributes estimate based on an approximation to the covariance matrix. The advantage of the proposed method over the standard method is that the length of the window size for the covariance computation is adaptively adjusted with the help of the instantaneous frequencies from the different components. Furthermore, since the polarization attributes are estimated for each time, no interpolation is needed. With the proposed method one can also estimate the magnitude and direction of the third polarization direction in the case of three-component record. However this estimate is not unique and is subject to the choice of the integer N that determines the size of the adaptive time window. The proposed method has potential application in wavefield separation and filtering of multicomponent seismic records using polarization analysis. Extension of the present method to the time-frequency domain will be considered in a forthcoming contribution to improve the polarzation analysis for dispersive waves such as Rayleigh waves in heterogeneous media.

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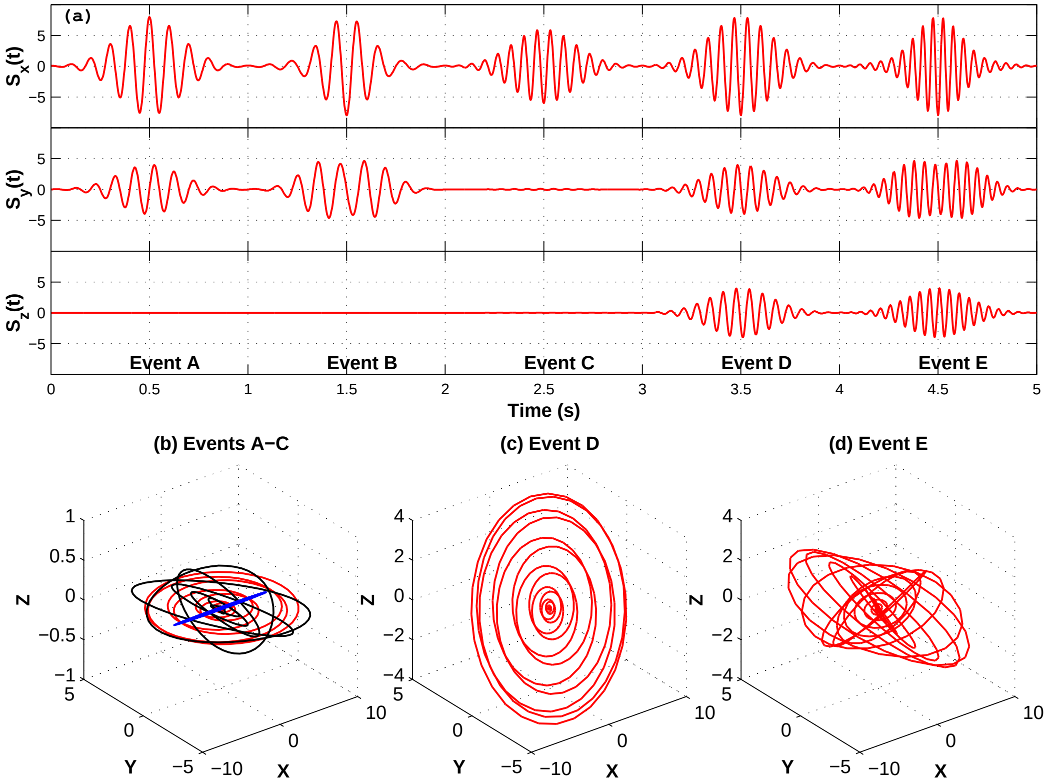
LIST OF CAPTIONS

Figure 1. (a) Three synthetic signals simulating a 3 component record with elliptically polarized events contained in the (x-z), (x-y) and (y-z) planes. Hodogram plots showing the ellipses in the different polarization plane for events A-C (b), for event D (c) and for event E (d).

Figure 2. 3D instantaneous polarization attributes in the time domain for x-y-z components: (a) instantaneous major, minor and second minor axis, (b) ellipticity ratio, (c) direction cosines of the planarity vector, (d) instantaneous frequencies. From the ellipticity ratios one can clearly identify the linearly polarized event ($\rho(t)$ close to zero) from those with elliptical or ellipsoidal polarization. The cosine directions allows an easier determination of the plane containing the polarization ellipse in space.



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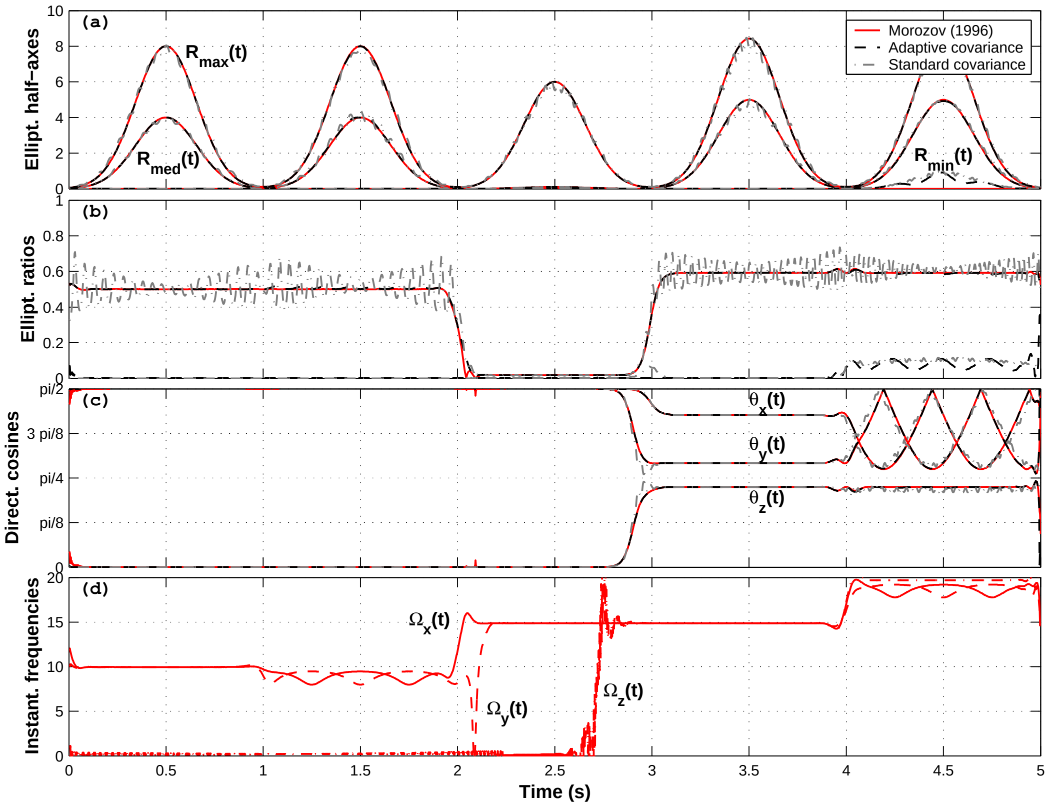

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